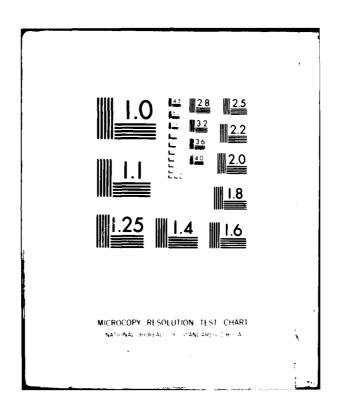
CITY COLL RESEARCH FOUNDATION NEW YORK F/G 19/1 AUTOMATED STRENGTH DETERMINATION FOR INVOLUTE FUZE GEARS.(U) AD-A117 593 FEB 82 6 G LOWEN, C KAPLAN DAAK10-79-C-0251 ARLCD-CR-81060 UNCLASSIFIED NL 1 1 3 Ars.



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which allow the determina- th the associated Lewis gear of involute gears including manner in which the root the involute and trochoid mes are discussed in detail. d.
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#### 1. IN TRODUCTION

This report describes the development and use of certain computer programs which allow the determination of relevant information with respect to strength calculations for all types of involute gears including fuze gears.

Both programs, LEWIS CIRCLES and LEWIS ENVELOPE, furnish the coordinates of the left side of any involute tooth (See Figure 1), together with the associated Lewis and AGMA geometry factors. The determination of these factors is based on the assumption that the maximum contact force F is applied at the tip of the tooth and that therefore the origin V of the inscribed Lewis parabola is located at the point of intersection of the line of action of this force and the tooth centerline. The point of tangency between this parabola and the tooth profile, together with the x-coordinate XTAN required for the determination of the above factors, is found by an iterative process.

The involute portion of the tooth profile is obtained in both programs by a mathematical description of the unwinding of the base circle tangent. The trochoid or root portion of the tooth profile is obtained by two different methods.

Program LEWIS CIRCLES finds the associated coordinates

by a simulation of the hob cutting or drafting process, while program

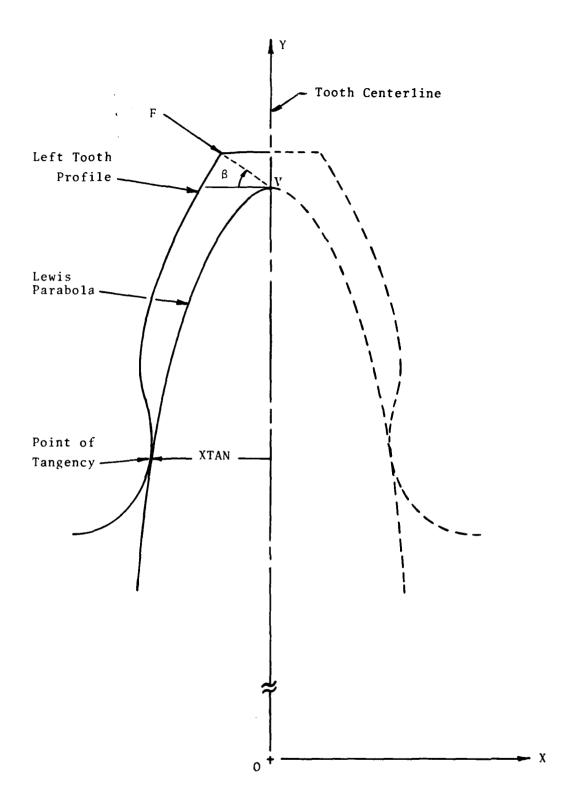


Figure 1. Tooth profile and Lewis parabola for maximum load  ${\bf F}$ 

LEWIS ENVELOPE makes use of an analytical method, based on the relationship between generating curves and envelopes. While the results of both programs are identical, program LEWIS CIRCLES cannot be used when the hob fillet radius is zero.

To assist the user as much as possible with the details of operation of these programs, the following outline of input and output parameters, is given: (The input and output parameters of both programs are identical).

### a. Input Data (All given for unity diametral pitch)

- 1. Number of Teeth
- 2. Effective distance (see Appendix B)
- 3. Unity diametrical pitch
- 4. Hob pressure angle
- 5. Circular tooth thickness at standard pitch circle
- 6. Hob fillet radius
- 7. Gear tooth addendum.
- 8. Angular increments for involute and trochoid generation

#### b. Output Data

1. Results of various intermediate computations

- 2. Lewis factor
- 3. AGMA geometry factor
- 4. Various parameters associated with the point of tangency between Lewis parabola and tooth profile
- 5. Coordinates of Tooth profile

As a further aid to the user, material on which this report is based is compiled in the appendixes.

### Appendix A: Determination of Involute Coordinates

This appendix gives the derivation of the x and y-coordinates of the involute portion of the tooth profile of an arbitrary tooth in terms of the coordinate system shown in Figure 1. Use is made of the angle  $\alpha$  which describes the unwinding of the base circle tangent.

## Appendix B: Determination of Path Coordinates of Effective Rack Point

After defining the concepts of effective rack point and effective distance for hobs with and without fillet radii, this appendix gives derivations for the path coordinates of the effective rack point for undercut and non-undercut gear teeth. In order to accomplish this, the generating angle  $\psi$ , which describes the rolling of the rack pitch line on the gear pitch circle, is introduced. Again, the results are given in terms of the tooth centerline coordinates shown in Figure 1.

### Appendix C: Generation of Tooth Root in Presence of Fillet Radius on Rack Cutter

Two methods of obtaining the coordinates of the tooth root are shown. The first method simulates the cutting or drafting process and is used in program LEWIS CIRCLES. The second method, which is based on certain relationships between generating curves and envelopes, forms the basis for the root generation in program LEWIS ENVELOPE.

### Appendix D: Initial and Final Angles for Trochoid and Involute Generation

This appendix first gives a review of existing procedures for the determination of the inner form radius for undercut and non-undercut teeth. Subsequently, expressions for the limits of the generating angles  $\alpha$  and  $\psi$  are derived for both types of teeth.

### Appendix E: Gear Strength Calculations. Determination of Lewis and AGMA Geometry Factors

Initially, the origin of the Lewis gear strength formula is reviewed and expressions for the determination of various associated parameters are derived. Subsequently, the iterative procedure for finding the point of tangency between the Lewis parabola and the tooth profile, which is used both in program LEWIS CIRCLES and in program LEWIS ENVELOPE, is explained.

### Appendix F: Program MATRIX

This appendix describes and lists program MATRIX, which is used to determine the size of the arrays in the two computer programs. It also contains sample outputs of program MATRIX

which pertain to the sample runs of both LEWIS programs. (See Section 3.)

### Appendix G: Program LEWIS CIRCLES

A complete listing of the program LEWIS CIRCLES is given, together with outputs of the two sample runs.

### Appendix H: Program LEWIS ENVELOPE

This appendix contains a complete listing of program LEWIS ENVELOPE. In addition, the results of the two sample runs are shown.

### 2. DESCRIPTION OF COMPUTER PROGRAMS

The following section gives descriptions of the essential steps of the programs LEWIS CIRCLES and LEWIS ENVELOPE. Program LEWIS CIRCLES is listed, together with sample runs, in Appendix G. Appendix H furnishes the same for program LEWIS ENVELOPE. The specific sample runs of both programs are discussed in Section 3.

The input and output parameters of both programs are identical and will therefore only be discussed in detail in conjunction with program LEWIS CIRCLES.

Since the vector Y and the matrix X, as used for the determination of the root profile in LEWIS CIRCLES (see also Appendix C-2: Iterative Method of Obtaining Root Profile), may be very large, it is necessary to use care in the dimensioning of these arrays as well as in the request for computer storage space.

Appendix F gives program MATRIX which allows the determination of the sizes of Y and X, as well as of the vectors Xl and Yl which are used for the initial storage of the involute coordinates.

### I. Program LEWIS CIRCLES (See Appendix G)

### Dimensioning of Arrays

(For more details on the use of the various arrays see description of computations below).

The vectors X1 and Y1 are used for the initial storage of the involute coordinates of the gear tooth (see Appendix A). The minimum number  $N_{Y1}$  of locations for either of these arrays is given by: (See also Appendix F)

$$N_{Y1} = \frac{\alpha_{fin} - \alpha_{in}}{\Delta \alpha}$$
 (1)

where

 $\alpha_{in}$  = initial involute generating angle [See equ. (D23) of Appendix D].

 $\alpha_{fin}$  = final involute generating angle [See equ. (D24)].

 $\Delta \alpha$  = increment of involute generating angle.

The arrays X2 and Y2 are used for the final storage of all values of the tooth profile coordinates. The required number of locations is represented by the sum of those needed for the vectors Y1 (or X1) and Y. It can only be determined once the length of the vector Y is known. The later is obtained with the help of the aforementioned program MATRIX, which also finds the size of matrix X.

### b. Input Parameters

The following parameters represent the required input data for the program. (They are also printed out on top of the first output page).

N = number of teeth in gear under consideration

BEFF =  $b_{eff}$  for unity diametral pitch (See Appendix B)

PD =  $P_d = 1.000$ , unity diametral pitch of hob

THETAD =  $\theta$ , the pressure angle of the hob (the D at the end expresses that an angle is given in degrees)

CAPTCS =  $T_{cs}$ , the circular tooth thickness of the gear at the standard pitch radius  $R_p = \frac{N}{2P_d}$ , in terms of  $P_d = 1.000$ 

 $RC = r_c$ , the rack fillet radius for unity diametral pitch

KADD = Addendum constant of gear, with the magnitude of the addendum defined by KADD/PD, in terms of  $P_d$  = 1.000

DELPSI =  $\Delta \psi$ , the increment of the roll angle of the rack pitchline on the pitch circle of the gear (See note below)

DELAL =  $\Delta\alpha$ , the increment of the involute generating angle(See note below)

CONB = Initial value of constant B of Lewis parabola[See equ. (E32)]

DELCON =  $\Delta B$ , the increment of the constant B of the Lewis parabola (See note below)

NRITE = Test constant. When NRITE = 1, all the tooth profile coordinates stored in X2 and Y2 are printed out. These coordinates are not printed out when NRITE = 0.

NOTE: The choice of the magnitudes of  $\Delta\alpha_{\bullet}$   $\Delta\psi$  and  $\Delta B$  is discussed in Section 3.

### c. Computations

Starting with line 29 of the program each location of the matrix X is filled with the value -10.00. This is necessary, since the subsequent sorting process must determine the largest value in rows whose locations are either unfilled or contain negative numbers. If such an unfilled location contains a zero, it will be found as the answer rather than the actual x-coordinate of the left root profile.

Subsequent to line 33 various gear parameters, such as the pitch radius  $R_{\rm p}$ , the base radius  $R_{\rm b}$  and the outside radius  $R_{\rm o}$  are computed. Further, the following quantities are determined:

B =  $b_{eff}$  =  $\frac{BEFF}{PD}$ , the actual distance to the effective point for the given diametral pitch

THETAB =  $\theta_b$ , see equ. (A1)

The later is computed with the help of the involutomery formula given by equ. (El6) of Appendix E. For this case:

$$T_1 = T_{cs}$$
,  $R_1 = R_p$ ,  $\phi_1 = \theta$ 

and

$$T_2 = T_b$$
 ,  $R_2 = R_b$  ,  $\phi_2 = 0$ 

so that

$$T_{b} = T_{cs} \frac{R_{p} \cos \theta}{R_{p}} - 2R_{b} [INV (0) - INV (\theta)]$$

or

$$T_b = T_{cs} \cos \theta + 2R_b [\tan (\theta) - \theta]$$

and finally,

$$\theta_{b} = \frac{T_{b}}{R_{b}} = \frac{T_{cs} \cos \theta}{R_{b}} + 2(\tan \theta - \theta)$$
 (2)

Following line 41 one finds:

EPS = 
$$\frac{\theta}{2}$$
 +  $\theta$ , for use in equ's. (B8) and (B9)

DELTA = 
$$\frac{b_{eff}}{R_{b} \sin \theta}$$
 -  $\tan \theta$ , see equ. (B22)

$$BALLOW = b_{effmax}$$
, according to equ. (B1)

Lines 45-47 represent a test which decides with the help of equ. (B1) whether or not the tooth is undercut.

Lines 48-60 are applicable for undercut teeth. The following computations are performed:

A = a, according to equ. (B2)

TAU = EPS, from line 42

The inner form radius RF =  $R_f$  is obtained with the help of equ. (D9). This non-linear algebraic equation is solved by subroutine DEKKER. The radii  $R_O$  and  $R_b$  represent the limit within which the root  $R_f$  is determined.

ALPHIN =  $\alpha_{in}$ , according to equ. (D23)

ALPHF =  $\alpha_{fin}$ , according to equ. (D24)

GAMT =  $\gamma_{tot}$ , according to equ. (D16)

PSIIN =  $\psi_{in}$ , according to equ. (D18)

PSIF =  $\psi_{fin}$ , according to equ. (D14)

Lines 62 to 82 are applicable to teeth which are not undercut. The essential computations are given by:

A = a, according to equ. (B10)

TAU = EPS + DELTA, from lines 42 and 43[This is n in equ's. (B22), (B24) and (B25)]

RF =  $R_f$ , according to equ. (D12)

GAMT =  $\gamma_{tot}$ , according to equ. (D16) with the above RF

ALPHIN =  $\alpha_{in}$ , according to equ. (D23). The test for  $\alpha_{in}$  has the purpose of avoiding an almost zero number when a zero should be obtained

ALPHF =  $\alpha_{fin}$ , according to equ.(D24) with the RF of line 66

PSIIN = 0, according to equ. (D20)

PSIF =  $\psi_{fin}$ , according to equ. (D19)

The computations represented by lines 82 to 88 are applicable to both types of teeth. In addition to converting certain angles from radians to degrees for printing out, the following parameters of the center of the rack fillet are computed:

- A2 =  $a_2$ , according to equ. (C1)
- B2 =  $b_2$ , according to equ. (C2)

Lines 89 to 104 concern themselves with the determination of the coordinates of the involute portion of the tooth. It is at first most convenient to store XI =  $x_{inv}$  and YI =  $y_{inv}$ , which are obtained from equations (A5) and (A6), respectively, in the vectors X1(I) and Y1(I). For I = 1, the angle  $\alpha = \alpha_{in}$ . When this counter has attained its maximum value, the angle  $\alpha$  is just a little larger than  $\alpha_{fin}$ . Note also that this maximum value of the counter I is stored as IPREV for future use. For purposes of subsequent computations it is desirable to have the counter of the coordinate locations of the tooth profile start at the tip of the tooth and end at that root location which corresponds to  $\psi_{fin}$ . For this reason, the sequence of the involute coordinate counter is inverted in the operations of lines 100 to 103. These coordinates are now stored in the vectors X2 and Y2. (Note that the counter

number corresponding to IPREV now denotes the location of the involute coordinates associated with the angle  $\alpha_{in}$ ).

The determination of the coordinates of the tooth root, i.e. the trochoid, starts in line 105. Initially, the vector Y(I) and the matrix X(I,J) are created in the manner described in Appendix C-2. Subsequently, a search procedure determines the largest value of the x-coordinate X2 associated with a given y-coordinate Y2. It is to be recalled that, the parameters appropriate to the type of tooth under consideration have already been stored and that, therefore the same basic expressions can be used in the program whether the tooth is undercut or not.

The following explains the manner in which the program fills the vector Y(I) and the matrix X(I,J):

- (1) The angle  $\psi$  is made equal to  $\psi_{in}$  and the counter I, for the y-coordinate, and the counter J, for the number of the circle, are initialized.
- (2) Starting with location 25 (line 111), the fillet center coordinates  $XC = x_c$  and  $YC = y_c$  are computed according to equ's. (C3) and (C4) or equations (C5) and (C6). Further, the y-coordinate  $YT = y_{tr}$  of the effective point of the rack is obtained with the help of equ. (B9) or

equ. (B25). If these computations were made for the first position of the fillet circle, i.e. for J=1 (in the program I=1 is used), control is transferred to location 80.

(3) Starting with location 80, the value for YT is multiplied by 1000 and the result is truncated such that the integer IYT results. This operation, which in essence multiplies a y-coordinate given to one one thousandth of an inch by 1000, is necessary for subsequent identification purposes. In addition, IYT is stored in vector Y(I) in location 90. (Thus, rather than storing the actual y-coordinate its thousand-fold value is used). If the the effective point of the first circle is involved I = 1. With the above accomplished, IYT is again made into a real number and divided by 1000 ( giving the y-coordinate to the nearest one one thousandth of an inch) for further work. It is now labelled YTTT [The y of equ. (C7)]. In order to be able to stop the computations for points on a given fillet circle, once

$$y \leq y_c - r_c , \qquad (3)$$

the test of lines 127 and 128 has been devised.

The x-coordinate associated with YTTT is then obtained

with the help of equ. (C9) of Appendix C and stored in X(I,J), where J is the circle number and I is the counter of the y-coordinate. This sequence, which starts on line 124 and ends with line 132 is continued until the limit of equ. (3) is reached for a given circle. In each step the counter I is incremented by 1, the integer IYT is decreased by 1 and stored in Y(I), and finally the associated x-coordinate is stored in the appropriate X(I,J). When the computations for any fillet circle are completed, control is transferred to location 150.

- (4) Starting with location 150, the stage is set to compute the x and y coordinates of points on subsequent circles: The angle  $\psi$  is decremented by  $\Delta \psi$ , and a test is made to check whether  $\psi_{\text{fin}}$  has been reached. In addition, the circle counter J is incremented by 1 as long as  $\psi < \psi_{\text{fin}}$ . To obtain the data for a new circle, control is transferred to location 25 again. In case the limiting angle  $\psi_{\text{fin}}$  has been reached, the value J corresponding to the last circle is stored as <u>JFIN</u>, and control is given to location 500 to commence the search for the largest value of X(I,J) in a given row of the array.
- (5) If control was transferred to location 25, the steps described in (2)above are repeated for the new position of the fillet circle. In order to be able to form an array

as shown in Table C-1 of Appendix C, it is necessary to establish that location Y(I) of any previous circle which contains an IYT of the same magnitude as that which corresponds to the effective point of the new circle.

This is accomplished by the **sorting** procedure starting in line 115 and ending in line 121: The counter I, which serves to call on the already existing Y(I), is initialized and YT is made into the integer IYT. The desired location Y(I) is found by comparing the content of each of the existing Y(I) with the value of the above IYT. When the test

$$Y(I) - IYT \leq 0 (4)$$

is met, control is transferred to location 90 and the present values of I and IYT are reestablished as the counter and the content of this Y(I).

At this point all operations for a given circle may be performed as described beginning with the second paragraph of section (3) above.

(6) As stated earlier, once all the possible Y(I) and X(I,J) are filled, control is transferred to location 500 to

search for the x-coordinates of the root profile.

The preparation for the determination of the largest value of X(I,J) corresponding to any y-coordinate Y(I) starts in location 500 (line 139). YFIN, the smallest value of the y-coordinate for which X(I,J) has been found earlier, is obtained by way of YCFIN. This variable is obtained with the help of the fillet center expressions (C4) or (C6), evaluated at  $\psi = \psi_{\mbox{fin}}$ . Then, similar to equ. (3):

$$YFIN = YCFIN - RC$$
 (5)

Subsequently, the following operations are performed:

- (1). The counter I is initialized. It is used to keep track of the Y(I) and X(I,J) of before.
- (2). The counter K = I + IPREV is introduced in location 501. This counter makes it possible to add the final values of the root coordinates to vectors X2 and Y2, which presently only contain the involute coordinates in locations from I = 1 to I = IPREV. (See line 98).

- (3). KFIN = K is introduced, in order to be able to label the last values of X2 and Y2.
- (4). The sorting process for each row of the array X(I,J) starts in line 144 with the initialization of X2(K) = -10.00. The loop between line 145 and 147 starts by comparing the above value with that of the content of X(1,1) i.e. the first value in the first row. If X(1,1) is larger than -10.00, X2(K) is made equal to X(1,1). If it contains the initially read-in value of -10.00, X2(K) remains as -10.00 (See line 31). In the next comparison, the content of X(1,2) is compared with the current value of X2(K). In case X(1,2) > X2(K), this value of X(1,2) is made the current value of X2(K). This sorting process is repeated until J = JFIN, the number of the last fillet circle (See line 137).
- (5). The outer loop of the sorting process includes
  lines 142 to 151. Once the largest value of any
  row of X(I,J) has been stored in X2(K), the content
  of the associated Y(I) is divided by 1000.00 and
  stored in Y2(K). This operation is repeated until
  the last value of Y2 becomes equal to or smaller
  than the previously determined YFIN.

Control is transferred to location 21, whenever the vectors X2 and Y2 contain a complete set of coordinates for the left tooth profile.

The determination of the various parameters, needed for the computation of the Lewis factor, starts in location 21 with the following:

PHIRO = 
$$\phi_{R_O}$$
, according to equ. (E22)

BETA =  $\beta$  , according to equ's. (E21)

THETAO =  $\theta_{R_O}$  , see equ. (E29)

RV =  $R_v$  , according to equ. (F28)

The search for the point of tangency between the Lewis parabola and the tooth profile is conducted according to the outline in Appendix E-4. It is first necessary to find the value of the counter K, used in X2(K) and Y2(K), which corresponds to that of the y-coordinate RV of point V of Figure E-4. The range between this point and the already known YFIN, for which K = KFIN, is the range in which the tangency will occur.

For computational reasons, the point START rather

than point V is chosen. Thus,

$$START = RV - .005 (inches)$$
 (6)

The location of that Y2 which corresponds to START is found in the loop between lines 165 and 169. Once the number of this location is found, it is labelled BEGIN.

The determination of the tangency point is carried out in the loop between lines 172 and 179. Using the counter J, the value of K is varied between

$$K = BEGIN + J - 1 \tag{7}$$

and

$$K = BEGIN + KFIN - 1 , (8)$$

where KFIN is the difference between the KFIN of line 143 and BEGIN of line 170.

The operation consists of the following steps:

(1). The value of XPAR =  $x_{par}$  is computed according to

equ. (E32), using the initial value of CONB = B [See equ. (E31)], for the full range of values of Y2(K). In each step, the difference

$$DIFF2 = |X2(K)| - XPAR$$
 (9)

is computed [See also equ. (E33)]. If this difference is less than or equal to zero, the point of tangency has been found and control is transferred to location 109 (line 180).

(2). If the point of tangency cannot be found for a given value of CONB, this value is decremented by DELCON (see line 178) and equ. (9) is used to test a new series of XPAR values. This process is continued until the values of X2(K) and Y2(K), associated with the tangency point, are found.

Finally, with  $X2(K_{tan})$  and  $Y2(K_{tan})$  found, EX = x is computed according to equ. (E35), and the Lewis factor as well as the AGMA geometry factor are determined with the help of equ's. (E11) and (E36), respectively.

### d. Output Parameters

(All lengths are given in inches and all angles in degrees).

The output of the program first states the numerical values of the input parameters N, BEFF, PD, THETAD, CAPTCS, RC, KADD, DELAL, DELPSI, DELCON and CONB. Subsequently, it is stated whether or not the tooth is undercut. In addition, the output gives numerical values for the following quantities:

RP = 
$$R_p = \frac{N}{2 P_d}$$
, the pitch radius of the gear

$$RB = R_{p} \cos \theta$$
 , the base radius of the gear

RO = 
$$R_p + \frac{KADD}{P_d}$$
, the outside radius of the gear

RF = 
$$R_f$$
, the inner form radius, either according to equ. (D9), or according to equ. (D12)

EPS = 
$$\frac{\theta}{2}$$
 +  $\theta$ , in degrees

THETAB = 
$$\theta_h$$
, according to equ. (2), in degrees

DELTA = 
$$\frac{b_{eff}}{R_{h} \sin \theta}$$
 -  $\tan \theta$ , according to equ. (B22), in

degrees

TAU = EPS or EPS + DELTA

GAMT =  $\gamma_{tot}$ , according to equ. (D16)

ALPHIN =  $\alpha_{in}$ , according to equ. (D23)

ALPHFIN =  $\alpha_{fin}$ , according to equ. (D24)

PSIIN =  $\psi_{in}$ , according to equ. (D18) or equ. (D20)

 $FSIFIN = \psi_{fin}$ , according to equ. (D14) or equ. (D19)

PHIRO =  $\phi_{R_0}$ , according to equ. (E22)

BETA =  $\beta$ , according to equ's. (E21)

CAPTO =  $T_o$ , tooth thickness at  $R_o$ , computed according to equ. (E16)

THETAO =  $\theta_{R_0}$ , computed according to equation (E15) and furnishing a result identical to that of equ. (E29)

RV =  $R_v$ , according to equ. (E28)

EX = x, according to equ. (E35)

YLEWIS = Y, the Lewis factor, according to equ. (E11)

YAGMA =  $Y_{AGMA}$ , according to equ. (E36)

TANGENCY K = value of counter K at point of tangency between

Lewis parabola and tooth profile

Y2(KTAN) = y-coordinate of tooth profile associated with point of tangency

X2(KTAN) = x-coordinate of tooth profile associated with
point of tangency

CONB = final value of constant B of Lewis parabola

NRITE = Test constant (see section b above)

If NRITE = 1, the program causes the printing of the tooth profile coordinates Y2(K) and X2(K) for all values of K, starting from the tip of the tooth down towards its root. The transition from the involute to the trochoidal part of the profile

occurs where the values of Y2(K) are given to one one-thousandth of an inch. Recall that, since the program computes the left tooth profile, the values of X2(K) are negative.

### II. Program LEWIS ENVELOPE (See Appendix H)

### a. Dimensioning of Arrays

Again, the vectors XI and YI are used for the initial storage of the involute coordinates of the gear tooth. The sizes of these vectors are also determined with the help of equ. (1), or by way of program MATRIX (see Appendix F).

The vectors X2 and Y2 must again hold the values of the involute coordinates as well as those of the root coordinates. The required dimensions of these arrays may be obtained by adding the number of rows of vector Y1 to the following number of trochoid computation intervals:\*

$$N_{t_r} = \frac{\psi_{in} - \psi_{fin}}{\Delta \psi} \qquad \text{(see below)}$$

where

 $\psi_{in}$  = initial trochoid generating angle [For undercut teeth use equ. (D18), for non-undercut teeth equ. (D20) is

The number of these intervals is identical with the column number for the tooth root interval, as given in the output of program MATRIX in Appendix F. See also equ. (F8).

applicable.  $\psi_{in}$  is positive or zero.]

- $\psi_{\mbox{fin}}$  = final trochoid generating angle. [For undercut teeth use equ. (D14), for non-undercut teeth use equ. (D19).  $\psi_{\mbox{fin}}$  is always negative. This explains the form of equ. (10).]
- $\Delta \psi$  = increment of roll angle of the rack pitch line (See earlier.)

#### b. Input Parameters

The input parameters for program LEWIS ENVELOPE are the same as those for program LEWIS CIRCLES.

#### c. Computations

With the exception that MATRIX X does not exist in this program, LEWIS ENVELOPE is parallel to LEWIS CIRCLES up to line 98: Various initial gear parameters are computed. It is decided whether or not the gear tooth is undercut and the associated parameters are determined. The involute coordinates are computed and stored in vectors X1 and Y1, with the last computation corresponding to the outside radius of the tooth. In order to be able to obtain

tooth profile coordinates which start with the tip of the tooth, the order of the vectors X1 and Y1 is again reversed and the result is stored in the vectors X2 and Y2. The location number of the last pair of involute coordinates (now associated with the angle  $\alpha_{in}$ ) corresponds again to the value of IPREV.

The determination of the tooth root coordinates starts in line 99 and proceeds in the following manner:

(1) The angle  $\psi$  is made equal to  $\psi_{\mbox{in}}$ , and the counter I is initialized at

$$I = IPREV + 1 \tag{11}$$

(2) A loop, starting with line 104 and ending with line 112, computes the coordinates XENV =  $x_{env}$  and YENV =  $y_{env}$  according to equ's. (C21) and (C22), respectively, until  $\psi = \psi_{fin}$  (PSIFD). The results are stored in vectors X2 and Y2, between the locations I = IPREV + 1 and I = KFIN.

The computations for the LEWIS and AGMA factors start in line 114 (statement no. 21) and are identical to those of program LEWIS CIRCLES up to and including line 151.

#### d. Output Parameters

The output parameters of program LEWIS ENVELOPE are identical

to those of program LEWIS CIRCLES. The print-out of the tooth profile coordinates differs from that in program LEWIS CIRCLES somewhat. For the present case, it was possible to separate the involute coordinates from the trochoid coordinates by appropriate headings, i.e. INVOLUTE PLOT and TROCHIOD PLOT.

#### 3. SAMPLE RUNS OF PROGRAM LEWIS CIRCLES AND LEWIS ENVELOPE

This section first discusses the results of two identical sample runs which were made with each of the two programs. The first type of run involves a non-undercut 36 tooth gear. The second concerns itself with an undercut 12 tooth pinion.

Appendix G lists program LEWIS CIRCLES on pages G-2 to G-9. The associated output for the 36 tooth gear is given on pages G-10 to G-18, while that for the 12 tooth pinion covers pages G-19 to G-37.

Appendix H lists program LEWIS ENVELOPE on pages H-2 to H-9. The associated output for the 36 tooth gear is given on pages H-10 to H-14. There are two outputs for the 12 tooth pinion. The one with the larger angle increment  $\Delta\psi$  is given on pages H-15 to H-19, while that for the smaller increment  $\Delta\psi$  is reproduced on pages H-20 to H-32. Finally, the results of certain selected computations of LEWIS and AGMA factors are given in tabular form.

#### I. Sample Runs of Program LEWIS CIRCLES

#### a. 36 Tooth Gear

#### 1. Input Parameters (See also Section 2-Ib)

With one exception, all input data of this run are printed out on top of the first output page (See p. G-10).

N = 36 teeth

BEFF = 1.215 in.

PD = 1.0, diametral pitch

THETAD = 20 degrees

CAPTCS = 1.846 in.

RC = .04 in.

KADD = 1.425 in.

DELAL = .250 degrees

DELPSI = .125 degrees

CONB = 4.0

DELCON = .001

Further, on the bottom of the first output page:

NRITE = 1

#### 2. Array Dimensions

The output of the program MATRIX on p. F-12 gives the following minimum dimensions for the various arrays associated with the 36 tooth gear:

Length of rows for involute

vectors X1 and Y1:

94

Length of rows and columns

for tooth root arrays:

POW = 370, COL = 85

This leads to the minimum number

of locations for vectors X2 and Y2:

94 + 370 = 464

As a consequence of the above, the following dimension statements are included in the program of this run (see p. G-2):

X1(100), Y1(100)

Y(400) , X(400, 100)

X2(500), Y2(500)

### 3. Output Data (See also Section 2-Id)

The first output statement on page G-10 conveys the fact that the "TOOTH IS NOT UNDERCUT". The subsequent data block is useful for checking purposes:

RP = 18.000 in.

RB = 16.91447 in.

RO = 19.425 in.

RF = 17.11373 in.

THETAB = 7.58392 degrees

GAMT = 0.0 degrees

EPS = 23.79196 degrees

DELTA = -8.82052 degrees

TAU = 14.97144 degrees

ALPHIN = 8.82052 degrees

ALPHFIN = 32.35465 degrees

PSIIN = 0.0 degrees

PSIFIN = -10.6258 degrees

PSIRO = 29.4532 degrees (This is actually PHIRO acc. to Section 2-Id)

BETA = 28.56269 degrees

CAPTO = .60383 in.

THETAO = .89052 degrees

RV = 19.25831 in.

The first line of the lowest block of results on page G-10 deals with the essential output of the program, i.e.:

EX = .55278 in., the Lewis dimension x

YLEWIS = .3685, the Lewis factor

YAGMA = .36221, the AGMA geometry factor

The second line of this data block gives further values which are of interest for checking purposes:

TANGENCY K = 237, i.e the point of tangency between the tooth profile and the Lewis parabola was found by the program to occur at Y2(237)

Y2(KTAN) = 16.937 in., y-coordinate of tooth profile at point of tangency.

CONB = 1.809, final value of constant B of Lewis

parabola

The listing of the coordinates of the left tooth profile, i.e. of the values stored in the associated vectors X2 and Y2, begins on top of page G-11. Recall that the values of these coordinates are printed out in such a way that they start at the tip of the tooth and end at its root. The involute portion of the profile begins with K = 1 and ends with K = 96. While the first set of values of the trochoidal portion of the tooth profile at K = 97 is not specifically marked, it may be recognized by the fact that Y2(97) represents the first y-coordinate which is given in terms of one one-thousandth of an inch. This trochoidal portion of the profile ends with K = 460. Note that the point of tangency at K = 237, is well above the last point of the profile.

h. 12 Tooth Pinion(Appendix G shows only the output of this run.)

#### 1. <u>Input Parameters</u>

With the exception of the NRITE statement, all input data are printed again on top of page G-19:

N = 12

BEFF = 1.0526 in.

PD = 1

THETAD = 20 degrees

CAPTCS = 1.57079 in.

RC = .300 in.

KADD = 1.000 in.

DELAL = .250 degrees

DELPSI = .500 degrees

CONB = 4.0

DELCON = .001

Further, as shown on the botton of page G-19:

NRITE = 1

#### 2. Array Dimensions

(The program listing containing the dimension statement

given below is not reproduced in this report.)

The output of program MATRIX on page F-13 gives the following minimum array dimensions for the given pinion data (For rationale, see parallel section for the 36 tooth gear.).

X1(150) , Y1(150)

Y(995) , X(995, 52)

X2(1145), Y2(1145)

#### 3. Output Data

The first output statement on p. G-19 indicates that the "TOOTH IS UNDERCUT". The subsequent data block may be interpreted as discussed in section 2-Id and is parallel to the one shown for the 36 tooth gear. The first line of the lowest block of results again represents the essential output of the run, i.e.:

EX = .34446 in., the Lewis dimension x

YLEWIS = .22964, the Lewis factor

YAGMA = .2364, the AGMA geometry factor

The second line of this data block gives:

TANGENCY K = 729

Y2(KTAN) = 5.019 in.

X2(KTAN) = -.780016 in.

XPAR TAN = .78003 in.

CONB = 2.903

The listing of the tooth profile starts on p. G-20. The involute portion of the tooth profile begins with K=1 and ends with K=152. The trochoidal part of the profile extends from K=153 to K=1133. Note that the point of tangency, at K=729, is well above the last computed profile point.

#### II. Sample Runs with Program LEWIS ENVELOPE

#### a. 36 Tooth Gear

#### 1. Input Parameters

With the exception of CONB = 5.0, the input parameters of this run with program LEWIS ENVELOPE are identical with those used in the run with program LEWIS CIRCLES (See section 3-Ia). As before, these data are listed principally on top of the first output page of the program. This output page is reproduced on p. H-10 of this report. (Note that the format of the first output page of program LEWIS ENVELOPE is the same as that of the first output page

of program LEWIS CIRCLES.)

#### 2. Array Dimensions

A certain part of the results of program MATRIX may be used to determine the dimensions of the vectors X1 and Y1 as well as those of the vectors X2 and Y2. Thus, from p. F-12:

Length of rows for involute vectors X1 and Y1: 94

Since there is now one set of root profile coordinates for each position of the angle  $\psi$ , the number of rows of these coordinates is equal to the number of columns given by program MATRIX under the heading of "Number of rows and columns for tooth root arrays". [See equ's. (10) and (F8)]. Thus, the number of root coordinate pairs equals 85 according to p. F-12.

The minimum dimensions for X2 and Y2 then become: 94 + 85 = 179. Actually, the following larger dimensions were included into the program for this run (see program LEWIS ENVELOPE on p. H-3):

X1(180) , Y1(180)

X2(1000), Y2(1000)

#### 3. Output Data

The central output data block, starting with "TOOTH IS NOT UNDERCUT" and ending with RV = 19.25831 is identical with that obtained by way of program LEWIS CIRCLES, as shown on p. G-10. The first line of the lowest output data block again furnishes the principal results of the run. (Recall that the root profile coordinates are computed in a different manner in program LEWIS ENVELOPE.) Thus,

EX = .55277 in.

YLEWIS = .36851

YAGMA = .36221

These results are identical with those obtained by way of program LEWIS CIRCLES. The second line of this lowest block is useful for comparison purposes (see Section 4):

TANGENCY K = 117

Y2(KTAN) = 16.93642 in.

X2(KTAN) = -1.132901 in.

XPAR TAN = 1.132924 in.

CONB = 1.809

The listing of the tooth profile coordinates starts on top of page H-ll. As for the comparable run with program LEWIS CIRCLES, the involute portion of the profile is given from K = 1 to K = 96. The root part of the profile extends from K = 97 to K = 182.

#### b. 12 Tooth Pinion

Program LEWIS ENVELOPE was run twice for the 12 tooth pinion. The input data of the first run are identical with those used earlier in conjunction with program LEWIS CIRCLES (See section 3-Ib). In the second run the increment  $\Delta\psi$  was reduced to .05 degrees in order to observe the influence of the more closely spaced y-coordinates of the root profile.

#### 1. Input Parameters for Run with $\Delta \psi$ = .5 degrees

The input data for this run, as shown on p. H-15, are the same as those given in section 3-Ibl.

#### 2. Array Dimensions for Run with $\Delta \psi = .5$ degrees

The minimum array dimensions for this run may again be obtained in the manner discussed in section II-a2 above with the help of the results of program MATRIX on p. F-13.

Since the lengths of the rows of involute vectors X1 and Y1 must be at least 150,

X1(150) , Y1(150)

The number of columns for the root array X of program LEWIS CIRCLES was determined to be 52. Therefore,

X2(202), Y2(202)

#### 3. Output Data for Run with $\Delta \psi$ = .5 degrees

The central output data block of p. H-15 is identical with that obtained from program LEWIS CIRCLES (see page G-19). The principal results, as given in the lowest output block, are as follows:

EX = .34443 in.

YLEWIS = .22962

YAGMA = .23643

TANGENCY K = 177

Y2(KTAN) = 5.01112 in.

X2(KTAN) = -.781767 in.

XPAR TAN = .781767 in.

CONB = 2.903

These results are essentially identical with those obtained from program LEWIS CIRCLES. The difference in the y-coordinate of the tangency point is due to the different manner of computing the tooth root coordinates. (See discussion in section 4 below.) The listing of all tooth profile coordinates is given on pp. H-16 to H-19 of Appendix H.

#### 4. Output Data for Run with $\Delta \psi = .05$ degrees

The input/output data of this run with a reduced angular increment are listed on pp. H-20 to H-32 of Appendix H. The principal results are as follows:

EX = .34447 in.

YLEWIS = .22965

YAGMA = .23635

TANGENCY K = 385

. N	BEFF (in.)	CAPTCS (in.)	RC (in.)	KADD (in.)	Lewis Factor	AGMA Factor
12	1.000	1.5707 <b>9</b>	. 239	1.000	. 24189	. 24807
13	"	**	,,	11	. 25493	.25922
2.;	11	11	1.	,	. 33822	.32876
30	,	97	,.	**	. 36134	. 34784
45	"	**	**	11	. 39494	.37542
50	11	11	11	11	.40209	.38121
100	"	**	,,	11	.43683	.40976
300	"	* *	ę,	••	. 45828	.42337
12	.800	1.57079	. 304	.800	. 33517	. 32598
13	"	**	11	*1	. 34831	.33712
15	"	11	11	11	.37016	. 35521
45	,,	**	"	11	. 47858	.44417
300	11	71	11	į t	.53370	.48946
12	1.0526	1.57079	.300	1.000	. 22964	. 23640
13	11	**	11	11	.24321	. 24817

(For all runs: Diametral pitch is unity and hob pressure angle is  $20^{\circ}$ . All runs made with program LEWIS CIRCLES.)

Table 1
Selected Computations of Lewis and AGMA Geometry Factors

Y2(KTAN) = 5.02859 in.

X2(KTAN) = -.777906 in.

XPAR TAN = .777909 in.

CONB = 2.903

## III. <u>Selected Computations of the Lewis and the AGMA Geometry</u> Factors

Table 1 shows the results of selected determinations of the Lewis and the AGMA geometry factors. In all cases, the computations were made with the help of program LEWIS CIRCLES and a diametral pitch of one, together with a 20 degree hob pressure angle were used.

#### 4. DISCUSSION

This section first presents a discussion of certain differences in the results for the two sample gears of section 3, as obtained with programs LEWIS CIRCLES and LEWIS ENVELOPE. (Table 2 juxtaposes the essentials of these results.)

Subsequently, a number of special features and peculiarities of both programs are pointed out.

#### I. Comparison of Results for 36 Tooth Gear and 12 Tooth Pinion

When comparing the results of the two programs for a given set of data, it must be kept in mind that the point of tangency of the Lewis parabola is generally located in the root portion of the tooth profile and that the programs produce the root profile in different ways.

Program	N "	YLEWIS	YAGMA	Y2(KTAN) (inches)	X2(KTAN) (inches)		CONB
CIRCLES	36	.36852	.3622	16.937	1.13277	1.13278	1.809
$\Delta \psi = .125$							
ENVELOPE	36	.36851	.36221	16.9364	1.13290	1.13292	1.809
$\Delta \psi = .125$							
CIRCLES	12	. 22964	.23640	5.0190	0.78002	0.78003	2.903
$\Delta \psi = .500$							
ENVELOPE	12	.22962	.23643	5.0111	0.78173	0.78177	2.903
$\Delta \psi = .500$							
ENVELOPE	12	. 22965	. 23635	5.0286	0.77791	0.77791	2.903
$\Delta \Psi = .050$		(					

Table 2
Comparisons of Results of Programs
LEWIS CIRCLES and LEWIS ENVELORA

Program LEWIS CIRCLES furnishes one x-coordinate of the tooth root for every one one-thousandth of an inch of the y-coordinate. The accuracy of these x-coordinates improves as the increment  $\Delta\psi$  is made smaller, i.e. as more circles become involved in this simulation of the drafting process. (See Appendix C-2.)

Program LEWIS ENVELOPE produces accurate x-coordinates of the root profile, because of its analytical origin. To obtain these x-coordinates in one one-thousandth of an inch increments, the increment  $\Delta \psi$  has to be quite small.

Let the results in Table 2 for the 36 tooth gear be compared first: Both sets of results were obtained with  $\Delta \psi$  = .125, and they are essentially identical. The spacing of the y-coordinates due to program LEWIS ENVELOPE may be seen on p. H-13 of Appendix H. The point of tangency is at Y2(117) = 16.9364 in. The preceding as well as the succeding values of the y-coordinates are approximately .006 inches away. Because of the magnitude of these distances, it is probably fortuitous that such close agreement could be obtained.

While the results for the Lewis and AGMA factors as well as for the constant B are also essentially identical in the

case of the 12 tooth pinion, there are some small differences in the x and y-coordinates of the root profiles. Even though the increment  $\Delta\psi$  = .500 is quite large, it had to be used in running program LEWIS CIRCLES, since the associated array X already had a 995 x 52 dimension and with that required a very large amount of computer core. (See output of program MATRIX on p. F-13.) When this increment was used to run program LEWIS ENVELOPE, the magnitude of the spacing of the y-coordinates near the point of tangency was approximately .020 inches. (See K = 177 on p. H-19.)

When program LEWIS ENVELOPE was run with  $\Delta \psi$  = .05. the spacing of the y-coordinates surrounding the point of tangency was of the order of .002 inches (See K = 385 on p. H-27.). This result appears to be the best of the three.

Aside form these small discrepancies, both programs furnish excellent results.

#### II Special Features of Program LEWIS CIRCLES

a. All input data must be expressed in terms of unity

diametral pitch.

- b. The program cannot be run for a hob fillet radius RC = 0, since its simulation of the drafting process requires such a radius. This limitation can be overcome by choosing a very small radius for the case of a sharp cornered hob. (By letting RC = .040 in. for the unity diametral pitch computation, an actual gear of 60 diametral pitch will be cut by a cutter with a radius of .00067 in.)
- The angular increment  $\Delta\alpha$  must be chosen sufficiently small if one desires a fairly close spacing of the y-coordinates of the involute part of the tooth profile for drawing purposes. A  $\Delta\alpha$  = .25 degrees caused a spacing of between .015 and .030 inches for the 36 tooth gear and a spacing of between .003 and .015 inches for the 12 tooth pinion. The larger spacing will always occur near the tip of the tooth.
- d. The choice of the root angular increment is especially important. The smaller  $\Delta\psi$ , the larger the number of circles for a given y-coordinate of the root region,

and with that the greater the accuracy of the resulting root profile. This becomes crucial when the hob fillet radius RC is small. For RC = .040 in. an increment  $\Delta \psi$  = .125 degrees gave good results. With RC = .300 in., the increment  $\Delta \psi$  = .500 deg. was satisfactory. (There is a trade-off between accuracy and core space requirement.)

Because the number of overlapping circles decreases near the bottom of the root profile, it is a good idea to check whether the y-coordinate of the point of tangency is at least one quarter of the magnitude of the hob fillet radius away from the final y-coordinate of the root profile. If this is not the case, it is hest to diminish the angle PSIFIN, i.e. continue further with the drafting simulation. (The above serves only as a cautionary note, since during all trial runs the point of tangency was well away from the end of the tooth root.)

e. Since the root profile coordinates are stored at

.001 inch intervals of the y-coordinate, the point of tangency will be accurate as long as the root profile is accurate.

#### III. Special Feature of Program LEWIS ENVELOPE

- a. All input data must also be expressed in terms of unity diametral pitch
- b. The program  $\underline{can}$  be run for all values of the hob fillet radius, including RC = 0.
- c. The above remarks concerning the angular increment  $\ensuremath{\Lambda\alpha}$  also apply here.
- d. While the y-coordinates of the root portion of the tooth profile are given at intervals of .001 in. by program LEWIS CIRCLES, the spacing of these coordinates by program LEWIS ENVELOPE depends entirely on the magnitude of the angular increment Δψ. If this increment is too large, the y-coordinates of the root profile may have a fairly wide spacing.

Since the point of tangency of the Lewis parabola can only be found for existing y-coordinates, this point will only then be accurately determined if the spacing of these coordinates is of the order of .001 inches. Since there is no core problem with program LEWIS ENVELOPE,  $\Delta\psi \leq .05$  degrees can readily be used.

#### APPENDIX A

#### DETERMINATION OF INVOLUTE COORDINATES

Figure A-l shows an involute gear tooth, originating at its base circle, in the standard vertical position as defined for this report.

It is assumed that the circular tooth thickness  $t_b = \overline{VV}'$  at the base circle and the associated angle  $\theta_b$  are known. To determine the x and y coordinates of the arbitrary point T on the involute, the unit vectors  $\overline{n}_{ou}$  and  $\overline{n}_{ut}$ , in directions  $\overline{OU}$  and  $\overline{UT}$ , respectively, must first be determined. Thus,

$$\overline{n}_{ou} = \cos(90 + \frac{\theta_b}{2} - \alpha) \overline{i} + \sin(90 + \frac{\theta_b}{2} - \alpha) \overline{j}$$
 (A1)

where

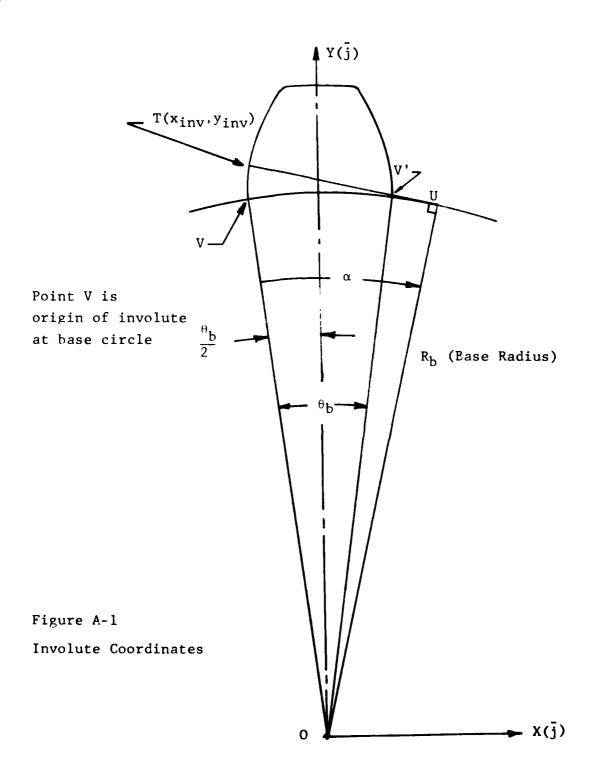
 $\alpha$  = the roll angle of the generating tangent  $\overline{\text{UT}}$ 

$$\theta_b = \frac{t_b}{R_b}$$
 , where  $R_b$  represents the base radius

Equation (Al) may be simplified to:

$$\overline{n}_{ou} = -\sin(\frac{\theta}{2}b - \alpha) i + \cos(\frac{\theta}{2}b - \alpha) \overline{j}$$
 (A2)

The unit vector  $\overline{n}_{ut}$  is found by cross multiplication of  $\overline{n}_{ou}$  by the unit vector  $\overline{k}\colon$ 



A-2 -60-

$$\overline{n}_{ut} = \overline{k} \times \overline{n}_{ou} = -\cos(\frac{\theta_b}{2} - \alpha) \overline{i} - \sin(\frac{\theta_b}{2} - \alpha) \overline{j}$$
(A3)

The vector  $\overline{\text{OT}}$  may now be written in the following form:

$$\overline{OT} = R_b \overline{n}_{ou} + R_b \alpha \overline{n}_{ut}$$
 (A4)

Substitution of equations (A2) and (A3) into the above and subsequent separation into x and y-components, gives the following coordinates of an arbitrary point T on the left involute profile:

$$x_{inv} = R_b \left[ -\sin \left( \frac{\theta_b}{2} - \alpha \right) - \alpha \cos \left( \frac{\theta_b}{2} - \alpha \right) \right]$$
 (A5)

and

$$y_{inv} = R_b \left[ \cos(\frac{\theta_b}{2} - \alpha) - \alpha \sin(\frac{\theta_b}{2} - \alpha) \right]$$
 (A6)

#### APPENDIX B

DETERMINATION OF COORDINATES OF PATH OF EFFECTIVE RACK POINT

In order to be able to plot the root of a gear tooth, as cut by a rack cutter, it is first necessary to obtain the x and y coordinates of the trochoidal path described by the effective point of the rack cutter tooth.

Figure B-l shows the effective points of rack cutter teeth with and without fillet radii as well as the associated effective

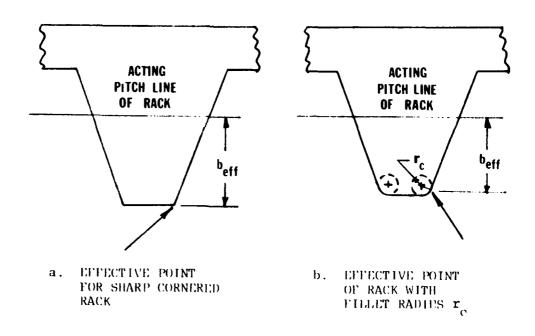


Figure B-1

Effective Point and Effective Distance  $b_{\mbox{eff}}$  of Racks with and without Fillet Radii

distances  $b_{\mbox{eff}}$ . When there is no fillet radius, (see Fig. B-la) the corner becomes the effective rack point and  $b_{\mbox{eff}}$  is represented by the distance from the acting pitch line of the rack to this corner. If the rack has a fillet, the point of tangency of the rack flank and the fillet circle becomes the effective point and  $b_{\mbox{eff}}$  is measured to this point.

The magnitude of  $b_{\mbox{eff}}$  decides for a given gear or pinion whether the teeth are undercut. Figure B-2 illustrates the applicable criterion.

If the effective point of the rack cutter tooth moves horizontally along a line which passes through point U(the origin of the involute) or above it, the involute tooth will not be undercut. Thus the limiting distance  $\frac{b}{cfmax}$  is given by

$$b_{effmax} = R_p - R_b \cos \theta$$
  
 $b_{effmax} = R_p (1 - \cos^2 \theta) = R_p \sin^2 \theta$  (B1)

Thus, generally,

If  $b_{eff} \leq R_p \sin^2 \theta$ , the gear tooth will not be undercut. The effective point of the rack initially contacts the gear tooth-to-be-cut above its base circle.

If  $b_{eff} > R_p \sin^2 \theta$ , the gear tooth <u>will be</u> undercut. The effective point of the rack initially contacts the tooth-

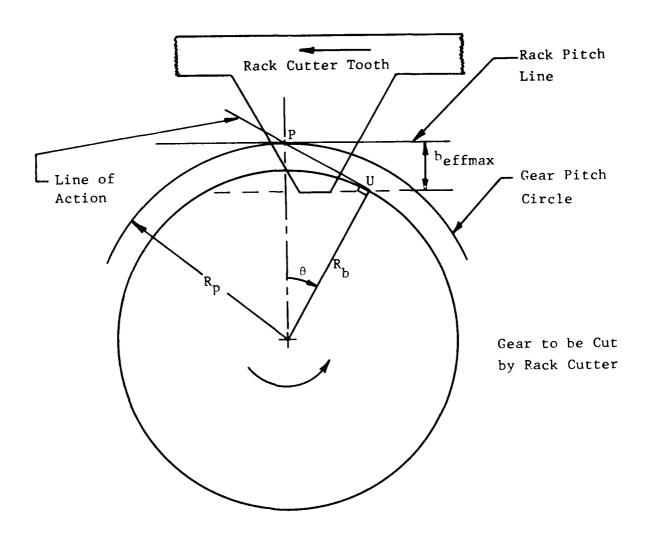


Figure B-2  $\begin{array}{ll} \mbox{Maximum Allowable $b$} \mbox{eff $w$} \mbox{hich} \\ \mbox{avoids Undercutting} \end{array}$ 

to-be-cut below its base circle.

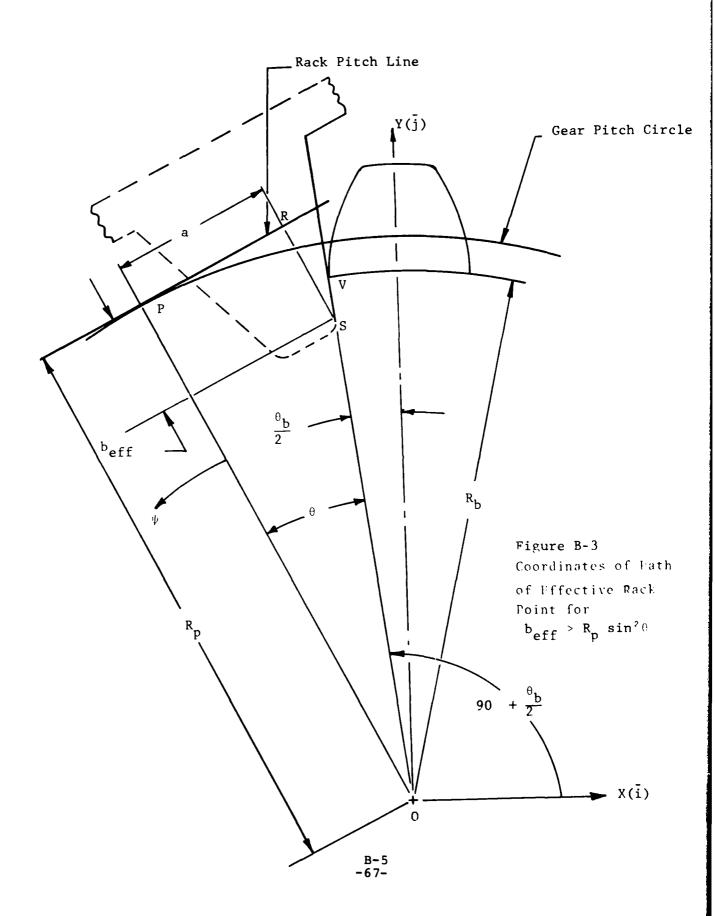
The following first gives derivations for the trochoid coordinates of the path of the effective rack point for both of the above cases. Appendix C shows how to obtain the associated coordinates of the path of the center of the fillet circle.

# 1. COORDINATES OF PATH OF EFFECTIVE RACKPOINT $\text{FOR } b_{\mbox{eff}} > R_p \ \sin^2 \theta$

Figure B-3 shows a rack cutter, which is about to cut the root of the indicated tooth, in its standard position. The flank of the rack tooth is tangent to the initial involute point V.

The angle  $\psi$ , which defines the rolling of the rack pitch line on the gear pitch circle, is zero when line  $\overline{OP}$  is at the rack pressure angle  $\theta$  to the initial involute tangent  $\overline{OV}$ .

As the angle  $\psi$  increases in a CCW direction, a part of the undercut root of the gear tooth is generated. A clockwise decrease in the angle  $\psi$  causes the completion of this root. In actual gear hobbing, it would also cause the generation of the involute portion of the tooth. The determination of the required maximum positive and negative values of the angle  $\psi$ , which are necessary to form a sufficient portion of the root profile, together with the determination of the inner form radius of undercut teeth, is shown in APPENDIX C.



When  $b_{eff} > R_p \sin^2 \theta$ , the distance from the instant center P to point R, becomes for the indicated standard position, i.e. for  $\psi = 0$ 

$$a = (R_p - h_{eff}) \tan \theta$$
 (B2)

For non-zero values of  $\psi$ , the distance  $\overline{PR}$  is given by

$$PR = a + R_{p} \psi \tag{?3}$$

To define the position vector  $\overline{OS}$ , from the center of the gear to the effective rack point S, it is necessary to define unit vectors for the directions of lines  $\overline{OP}$  and  $\overline{PR}$ , respectively.

$$\bar{n}_{op} = \cos(90 + \frac{\theta_b}{2} + \theta + \psi)\bar{i} + \sin(90 + \frac{\theta_b}{2} + \theta + \psi)\bar{j}$$
 (B4)

or

$$\bar{n}_{op} = -\sin(\frac{\theta_b}{2} + \theta + \psi)\bar{i} + \cos(\frac{\theta_b}{2} + \theta + \psi)\bar{j}$$
 (B5)

Further,

$$\bar{n}_{pr} = -\bar{k} \times \bar{n}_{op} = \cos(\frac{\theta b}{2} + \theta + \psi) \ \bar{i} + \sin(\frac{\theta b}{2} + \theta + \psi) \ \bar{j}$$
 (B6)

The position vector  $\overline{OS}$  may now be defined as:

$$\overline{OS} = R_{p} \overline{n}_{op} + (a + R_{p} \psi) \overline{n}_{pr} - b_{eff} \overline{n}_{op}$$
(B7)

Substitution of equ's (B5) and (B6) and separation of x and y components gives the following coordinates for the path of the effective rack point:

$$x_{tr} = -(R_p - b_{eff}) \sin(\frac{\theta_b}{2} + \theta + \psi) + (R_p \psi + a) \cos(\frac{\theta_b}{2} + \theta + \psi)$$
(B8)

and

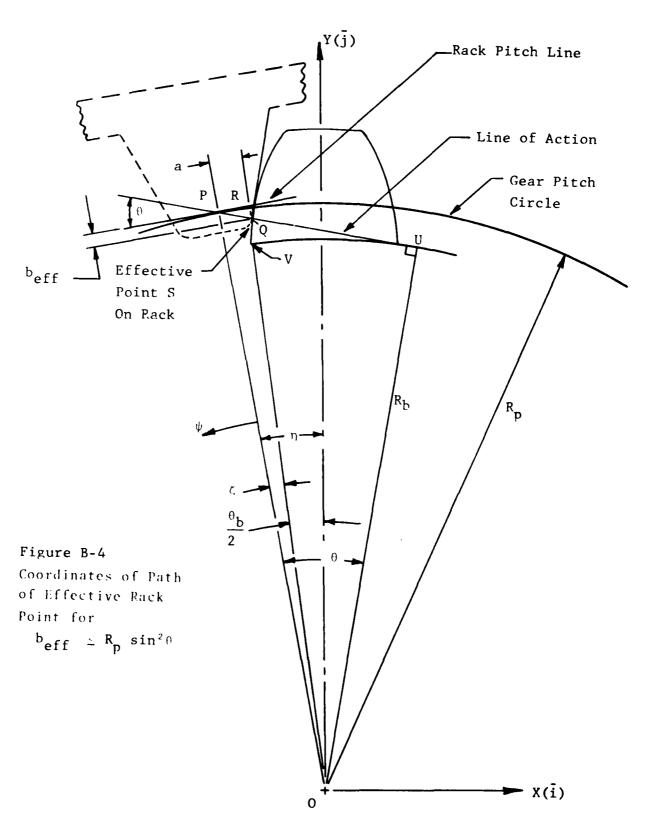
$$y_{tr} = (R_p - b_{eff}) \cos \left(\frac{\theta}{2} + \theta + \psi\right)$$

$$+ (R_p \psi + a) \sin \left(\frac{\theta}{2} + \theta + \psi\right)$$
(B9)

#### 2. COORDINATES OF PATH OF EFFECTIVE RACK POINT

FOR 
$$b_{eff} \leq R_p sin^2 \theta$$

Figure B-4 shows the effective point of the rack making initial contact with point Q on the (future) involute profile. (This is considered standard position for the non-undercut teeth.) This point must be located on the line of action  $\overline{\text{UP}}$ . Note that, point V again represents the initial point of the involute. The angle  $\psi$ , which again defines the rolling of the rack pitch line on the pitch circle of the gear is zero in the indicated standard position, when line  $\overline{\text{OP}}$  makes the rack pressure angle  $\theta$  with the instantaneous base radius  $\overline{\text{OU}}$  of length  $R_{b}$ . A clockwise decrease in  $\psi$  starts the generation of both the involute and root profiles. For a discussion of the limits of angle  $\psi$  see APPENDIX C. (Let the rack in Figure B-4 be considered to be in standard position.) For the present case, with  $\text{beff} \leq R_{p} \sin^{2}\theta$ , the distance a becomes:



B-8 -70-

$$a = \frac{b_{eff}}{\tan \theta}$$
 (B10)

This represents the distance  $\overline{PR}$  when  $\psi = 0$ . In general

$$\overline{PR} = a + R_{p} \psi \tag{B11}$$

To define the position vector  $\overline{OS}$ , from the gear center 0 to the effective rack point S, it is necessary to introduce unit vectors for the directions of lines  $\overline{OP}$  and  $\overline{PR}$ , respectively, Now, (see Figure B-4):

$$\bar{n}_{op} = \cos(90 + r_i + \psi)\bar{i} + \sin(90 + \eta + \psi)\bar{j}$$
 (B12)

or

$$\bar{n}_{op} = -\sin(\eta + \psi)\bar{i} + \cos(\eta + \psi)\bar{j}$$
 (B13)

further,

$$\bar{n}_{pr} = -\bar{k} \times \bar{n}_{op} = \cos(\eta + \psi)\bar{i} + \sin(\eta + \psi)\bar{j}$$
 (B14)

The angle  $\eta$  is defined as follows:

$$\eta = \frac{\theta_b}{2} + \zeta \tag{B15}$$

where

$$\zeta = \theta -$$
 VOU, (B16)

and  $\boldsymbol{\theta}$  is the pressure angle of the rack .

Further,

$$vou = \overbrace{\frac{vu}{R_b}} = \frac{\overline{q}\underline{u}}{R_b}$$
 (B17)

by the properties of the involute curve.

But

$$\overline{QU} = \overline{UP} - \overline{PR}$$
, (B18)

where

$$\overline{UP} = R_b \tan \theta$$
 (B19)

and

$$\overline{PR} = \frac{b_{eff}}{\sin \theta} . \tag{B20}$$

Finally,

$$VOU = \tan\theta - \frac{b_{eff}}{R_{b} \sin\theta}$$
 (B21)

and, according to equ's (B15) and (B16):

$$\eta = \frac{\theta}{2} + \theta + \frac{b_{eff}}{R_{bsin}\theta} - \tan\theta$$
 (B22)

The position vector  $\overline{\text{OS}}$  may now be written:

$$\overline{OS} = R_{p} \overline{n}_{op} + (a + R_{p} \psi) \overline{n}_{pr} - b_{eff} \overline{n}_{op}$$
 (B23)

Substitution of equ's (B13) and (B14) into the above, followed by the separation of x and y components leads to the following coordinates of the effective point for the present

case

$$x_{tr} = -(R_p - b_{eff})\sin(\eta + \psi) + (R_p\psi + a)\cos(\eta + \psi)$$
 (B24)

and

$$y_{tr} = (R_p - b_{eff})\cos(\eta + \psi) + (B25)$$

$$(R_p \psi + a)\sin(\eta + \psi)$$

#### APPENDIX C

## GENERATION OF TOOTH ROOT IN PRESENCE OF FILLET RADIUS ON RACK CUTTER

The present appendix shows two methods of obtaining the coordinates of a tooth root, i.e. the portion of the tooth below the involute profile, when the rack cutter has a fillet radius  $r_c$ . Now, the root profile is not generated by the effective point, as is the case for a sharp cornered rack, but it becomes the envelope of the various positions of the fillet circle. This fillet circle serves as a generating curve.

The first method simulates the applicable drafting method of finding an envelope by a purely numerical computer procedure. The center of the fillet circle is first found for a given position of the rack pitch line. Then, the x-coordinates of coints on the appropriate portion of the associated circle are found as functions of equally spaced y-coordinates. This process is repeated for other positions of the rack pitch line. For all angles  $\psi$  the x-coordinates of circle points, which are associated with certain y-coordinates , are stored in the same row of a matrix. If the angular increment of pitch line rotation is sufficiently small, that x-coordinate which is closest to the tooth center line represents the desired point on the root profile.

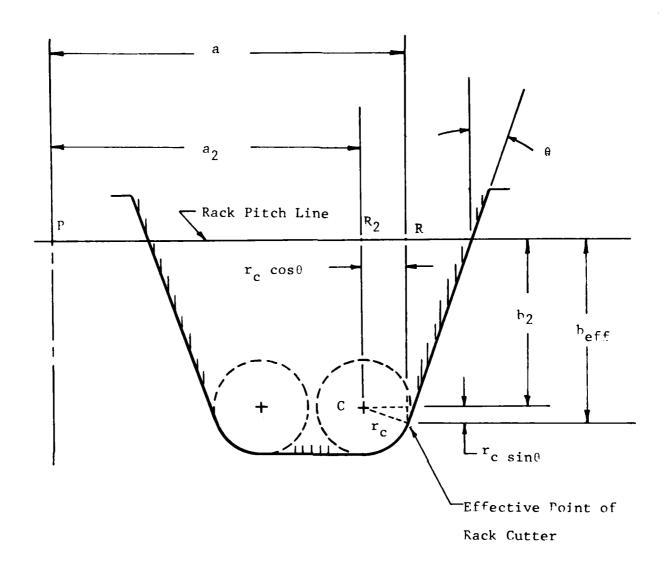


Figure C-1
Center of Fillet Radius in
Standard Position

The second method was first shown by R. G.

Mitchiner and H.H. Mabie in "The Determination of the Lew s

Form Factor and the AGMA Geometry Factor J for External Spur Gear

Teeth". Paper No. 80-DET-59, Design Engineering Technical

Conference, Beverly Hills, CA., Sept. 28 - Oct. 1, 1980. It

makes use of the fact that, the instantaneous centers of curvature of a generating curve and its envelope lie on a line

through the instant center of rotation of the two planes

involved. The contact point between the generating curve and

the envelope also lies on this line and it represents a point on

the final envelope profile.

Before either of these procedures can be shown in detail, it is necessary to find a method for determining the coordinates of the center of the fillet radius as a function of the angle  $\psi$ . This is accomplished by the appropriate adaptations of the expressions given in Appendix B.

# 1. Coordinates of Center Of Fillet Radius for Undercut and Non-Undercut Teeth

Figure C-1 shows a rack cutter tooth as it appears both for undercut and non-undercut gear teeth in the standard position of Figure B-3 and B-4, respectively. The distance from point R, on the rack pitch line, to the effective rack point is represented by  $b_{\mbox{eff}}$ . The distance  $\overline{\mbox{PR}}$ , from the instant center P to point R, which corresponds to the effective rack point,

is given by length a. [As before, either equ's. (B2) or (B10) are applicable.] If one wishes to express the position of the center C of the right hand fillet (of radius  $r_c$ ) with respect to the instant center P in this standard position, one obtains:

$$a_2 = \overline{PR}_2 = a - r_c \cos \theta \tag{C1}$$

and

$$b_2 = \overline{R_2C} = b_{eff} - r_c sin\theta$$
 (C2)

With the above it is now possible to adapt the coordinate expressions for the effective rack point to new ones for the fillet center C by replacing the distances a and  $b_{\mbox{eff}}$  by  $a_2$  and  $b_2$ , respectively. Thus, one obtains for undercut teeth, according to equations (B8) and (B9):

$$\mathbf{x}_{c} = -(\mathbf{R}_{p} - \mathbf{b}_{2}) \sin \left(\frac{\theta_{b}}{2} + \theta + \psi\right)$$

$$+ (\mathbf{R}_{p}\psi + \mathbf{a}_{2}) \cos\left(\frac{\theta_{b}}{2} + \theta + \psi\right)$$

$$\qquad \qquad \bullet$$
(C3)

and

$$y_c = (R_p - b_2) \cos (\frac{\theta_b}{2} + \theta + \psi)$$

$$+ (R_p \psi + a_2) \sin (\frac{\theta_b}{2} + \theta + \psi)$$
(C4)

Similarly, the coordinates of the fillet center C for <u>non-undercut</u> teeth become with the help of equ's. (B24) and (B25):

$$x_c = -(R_p - b_2) \sin (\eta + \psi) + (R_p \psi + a_2) \cos (\eta + \psi)$$
 (C5)

and

$$y_c = (R_p - b_2) \cos (\eta + \psi) + (R_p \psi + a_2) \sin (\eta + \psi)$$
 (C6)

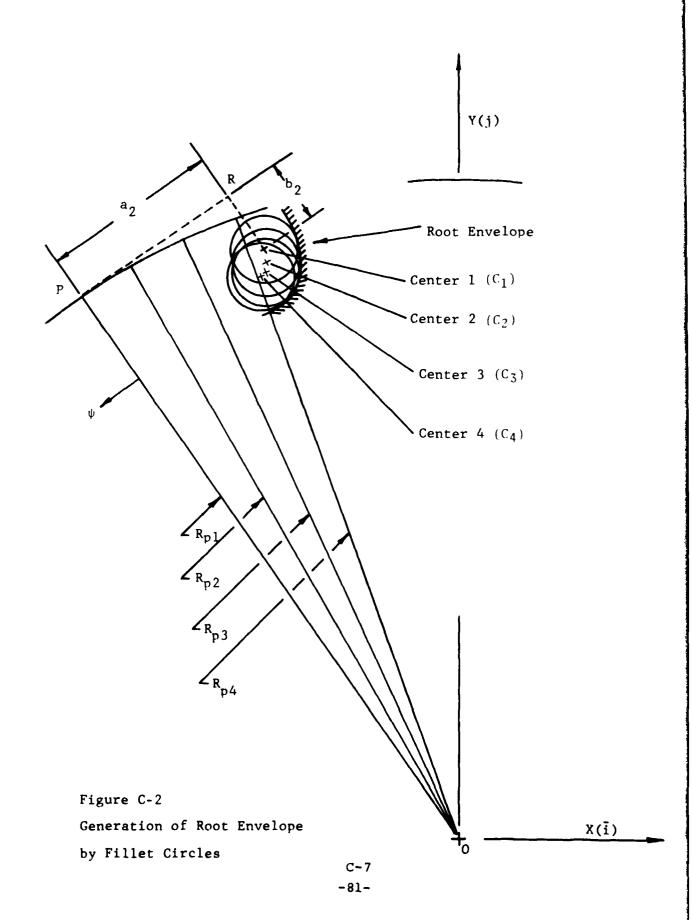
### 2. Iterative Method of Obtaining Root Profile

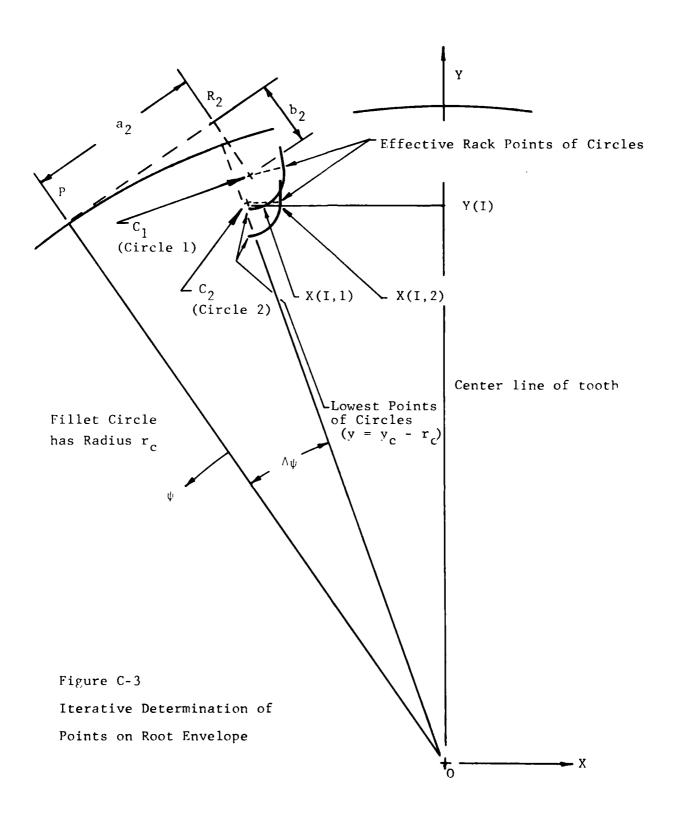
Figure C-2 indicates how successive fillet circles may be used to obtain the root profile envelope for undercut as well as non-undercut teeth. Point  $C_1$  represents the fillet center when the rack cutter is in the usual standard position. The operative pitch radius is  $\overline{OP} = R_{p1}$  and the coordinates of point  $C_1$ , with respect to point P, are given by  $a_2$  and  $b_2$  (as defined in the previous section). Centers  $C_2$ ,  $C_3$  and  $C_4$  result after successive clockwise rotations of the rack pitch line, with associated pitch radii  $R_{p2}$ ,  $R_{p3}$  and  $R_{p4}$ , respectively.

Figure C-3 represents an enlarged view of the relevant segments of two such fillet radii. It will serve to illustrate the following explanation of the computational procedure which is used to simulate the drafting method of obtaining the root envelope:

a. The coordinates  $\mathbf{x}_{\mathbf{c}}$  and  $\mathbf{y}_{\mathbf{c}}$  of centerpoint  $\mathbf{C}_{\mathbf{l}}$  of the first circle are computed with the help of cither equations (C3) and (C4) or equations (C5) and (C6), depending on whether the tooth is undercut or not.

The counter J is used to indicate the number of the circle. J = 1 for the present case





and generally, for that circle which corresponds to the intial angle  $\psi_{\mbox{in}}$ . The total number of circles depends on the increment  $\Delta\psi$  as well as on the final angle  $\psi_{\mbox{fin}}$ . (These initial and final angles are defined in Appendix D).

b. Since the center of the fillet circle never lies at the origin, the equation for points on the circle is given by:

$$(x - x_c)^2 + (y - y_c)^2 = r_c^2$$
 (C7)

The x-coordinates of points on this circle, for a given value of the y-coordinate, may be obtained from

$$x = x_c + \sqrt{r_c^2 - (y - y_c)^2}$$
 (C8)

For practical purposes it is only necessary to obtain the x-coordinates of points whose y-coordinates are located between the effective rack point of a given circle and  $y = y_c - r_c$  (See Figure C-3). (\*) Since the center of all

<sup>(\*)</sup> Points above the effective point on the rack flank are not involved in the generation of the final root profile.

generating circles lie in the second quadrant, their x-coordinates will always be negative.

Further, since the circle segments of interest will always be located to the right of their center points, the values of all x-coordinates will be larger than those of their associated  $x_c$ 's. For this reason, the positive sign must be used in equation (C8), i.e.:

$$x = x_c + \sqrt{r_c^2 - (y - y_c)^2}$$
 (C9)

c. The computation is continued by determining the y-coordinate of the effective rack point of circle 1 either with the help of equ. (B9) or equ. (B25), depending on whether the tooth is undercut or not. The y-coordinates are assigned the counter I and they will be stored in the vector Y(I). For this specific computation I = 1. The associated x-coordinate is then found with the help of equ. (C9), and its values is stored in the matrix X(I,J). Since this is the first x-coordinate of circle 1, it will be located by X(1,1). Subsequently, the first y-coordinate is decremented by .001 inch and its associated x-coordinate is again obtained with the help of equ. (C9). The value of the y-coordinate is stored in Y(2), while the x-coordinate belongs to X(2,1). This process is repeated until  $y = y_c - r_c$  (with the  $y_c$  of circle 1), filling the appropriate Y(I) and X(I,1).

d. The angle  $\psi$  is now decremented by  $\Delta \psi$ , as indicated by Figure C-3. J=2 for this second position of the fillet circle. Again, the coordinates  $\mathbf{x}_{\mathbf{C}}$  and  $\mathbf{y}_{\mathbf{C}}$  of point  $\mathbf{C}_{\mathbf{2}}$  are found, together with the y-coordinate of the effective rack point of circle 2. The later is identical to one of the y-coordinates of circle 1. In order to find the value of I which corresponds to this Y(I) of circle 1, all previously computed values of Y(I) are compared to the present one. (This sorting process is made possible by multiplying the values of all the y-coordinates stored in Y(I)

by one thousand and then truncating them to make a four digit integer out of them. (See Program description in Section 2). Once the specific value of I = f has been found, it is assigned to X(f,2) for storing the x-coordinate of the effective point of circle 2. The process of decrementing the y-coordinates of the second circle by .001 inch each time and computing the associated x-coordinates, until the lower limit of this circle has been reached, follows. It is identical to the one described for circle 1 in part (c) above. All values of these coordinates are assigned to the appropriate Y(I) and X(I,2).

(The above operations are repeated in the program many times. Table C-1, which shows a typical vector Y(I) and its associated matrix X(I,J) will be discussed below).

e. Figure C-3 shows two circle points on the horizontal line through Y(I). These are X(I,1) on circle 1 and X(I,2) on circle 2. Since X(I,2) is closest to the tooth center line, it becomes the accepted value for the x-coordinate

col.1	col.2	co1.3	col.4	co1.5	co1.6	col.7	co1.8
I	Y(I)	X(I,1)	X(I,2)	X(I,3)	X(I,4)	X(I,5)	X(I,6)
		J = 1	J = 2	J = 3	J = 4	J = 5	J = 6
		ψ = 0 <sup>0</sup>	$\psi = -3^{\circ}$	$\psi = -6^{\circ}$	$\psi = -9^{\circ}$	$\psi = -12^{\circ}$	$\psi = -15^{\circ}$
1	3.628	981					
19	3.610	977					
29	3.600	976					
49	3.580	974					
66	3.563	974	968				
104	3.525	978	963				
122	3.507	982	963	963			
169	3.460	-1.001	9 <b>69</b>	960	966		
206	3.423	-1.027	982	965	965	976	
231	3.398	-1.051	995	972	968	976	992
289	3.340		-1.045	-1.001	986	987	999
342	3.287			-1.053	-1.020	-1.013	-1.021
387	3.242				-1.073	-1.051	-1.054
421	3.208					-1.101	-1.094
446	3.183						-1.143
459	3.170			<u> </u>			-1.208

Table C-1

Typical Vector Y(I) and Matrix X(I,J)

(Diagonal lines indicate largest value in any row. All values in inches).

of the tooth profile associated with Y(I).

(In the actual program there are many X(I,J) values for a given Y(I). Again, that x-coordinate which is closest to the tooth center line becomes the desired point on the root profile).

Table C-1 Illustrates the relationship between the vector Y(I) and the matrix X(I,J) with selected numbers from an actual computer run. Column 1 contains the counter I which identifies the y-coordinates on the fillet circle segments. While I takes values from 1 to 459, in steps of one, in the actual run, the table shows only a limited number of y-coordinates. Column 2 contains these coordinates. Each one is common to all circle points on the same horizontal line. Columns 3 to 8 list the x-coordinates of these circle points. With  $\psi_{\rm in}=0^{\circ}$ ,  $\psi_{\rm fin}=-15^{\circ}$  and  $\Delta\psi=-3^{\circ}$ , the counter J, which identifies the circle number, ranges from 1 to 6. The first number in each column represents the x-coordinate of the effective point of the rack in the given position. (This was the basis of the choice of the I's in the table).

Once such a "table" has been completed by the computer, the x-coordinates of the root profile are determined by the largest

(i.e. least negative) number in any one row of the matrix X(I,J). For example for I=231, where y=3.398 in., the profile coordinate x=-.968 in.

## 3. Analytical Method of Obtain Root Profile Coordinates

Figure C-4 shows a rack with a fillet radius  $r_c$  which is generating the indicated root profile. Since the Euler-Savary Curvature Theory (\*\*) shows that, the contact point between a generating curve and its envelope lies on a line connecting the instant center P and the center of curvature C of the generating curve, point S represents a point on the root profile. To obtain the coordinates of this envelope point the components of the vector

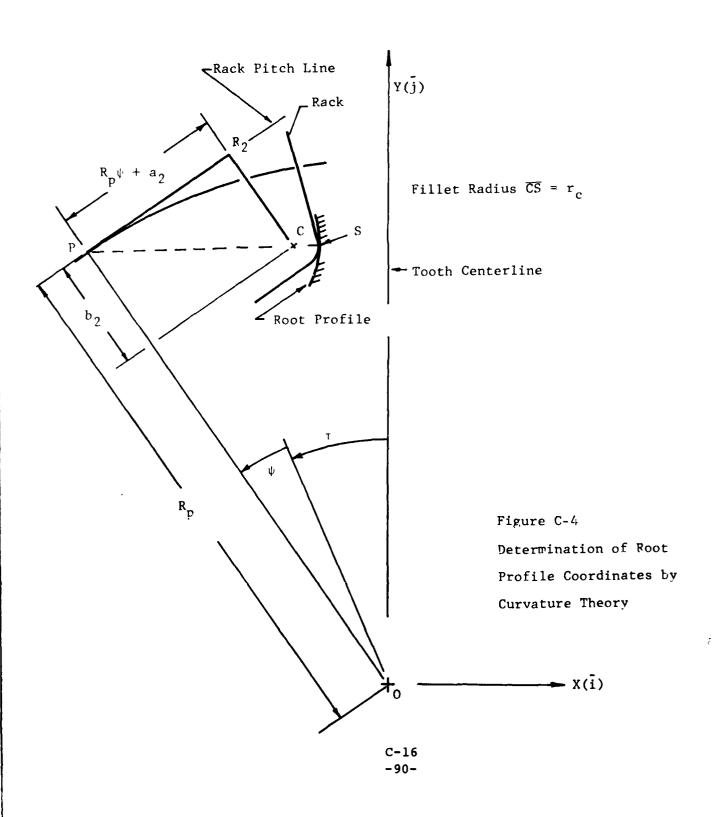
$$\overline{OS} = \overline{OC} + \overline{CS} = \overline{OC} + r_{c} \bar{n}_{cS}$$
 (C10)

must be determined. The unit vector  $\overline{n_{cs}}$  has the direction of line  $\overline{PC}$ , as indicated above.

The components of the vector  $\overline{OC}$  were given by equ's. (C3) and (C4) or by equ's. (C5) and (C6) for undercut and non-undercut teeth, respectively.

In order to obtain a single set of expressions, certain angles, found in the above equations, and which were first introduced

<sup>(\*\*)</sup> N. Rosenauer and A. H. Willis: Kinematics of Mechanisms, pp. 60-62, Dover Publications, Inc., No. S1796, New York, 1967.



in Appendix B, are now redefined as follows (see Figure C-4):

For undercut teeth, let

$$\tau = \frac{\theta_b}{2} + \theta \tag{C11}$$

[See also unit vectors  $\bar{n}_{op}$  and  $\bar{n}_{pr}$  for undercut teeth, as given by equ's. (B5) and (B6)].

For non-undercut teeth , let

$$\tau = \eta = \frac{\theta_b}{2} + \theta + \frac{b_{eff}}{R_{bsin\theta}} - \tan\theta$$
 (C12)

[Compare the above to the angles of the unit vectors of equ's. (B13) and (B14)].

With the above, equ's. (C3) to (C6) can be generalized to the following form.

For the x-component of vector  $\overline{OC}$ :

$$x_c = -(R_p - b_2) \sin (\tau + \psi) + (R_p \psi + a_2) \cos (\tau + \psi)$$
 (C13)

For the y-component of vector  $\overline{OC}$ :

$$y_c = (R_p - b_2) \cos (\tau + \psi) + (R_p \psi + a_2) \sin (\tau + \psi)$$
(C14)

To derive an expression for the unit vector  $\bar{n}_{cs}$ , it is first necessary to express the unit vector  $\bar{n}_{op}$  and  $\bar{n}_{pr}$ , alluded to earlier, in terms of the angle  $\tau$ . Thus,

$$\bar{n}_{op} = -\sin (\tau + \psi)\bar{i} + \cos (\tau + \psi)\bar{j}$$
 (C15)

and

$$\bar{n}_{pr} = \cos (\tau + \psi)\bar{i} + \sin (\tau + \psi)\bar{j}$$
 (C16)

Then, as Figure C-4 shows:

$$\tilde{n}_{cs} = \frac{\overline{PC}}{|PC|} = \frac{(PR)\bar{n}_{pr} - (RC)\bar{n}_{op}}{\sqrt{(PR)^2 + (RC)^2}}$$
(C17)

or, with the appropriate substitution:

$$\frac{(R_{p}\psi + a_{2})[\cos(\tau + \psi)\bar{i} + \sin(\tau + \psi)\bar{j}] - b_{2}[-\sin(\tau + \psi)\bar{i} + \cos(\tau + \psi)\bar{j}]}{\sqrt{(R_{p}\psi + a_{2})^{2} + b_{2}^{2}}}$$
(C18)

or

$$\bar{n}_{cs} =$$

$$[(R_{p}\psi + a_{2})\cos(\tau + \psi) + b_{2}\sin(\tau + \psi)]i + [(R_{p}\psi + a_{2})\sin(\tau + \psi) - b_{2}\cos(\tau + \psi)]i$$

$$\sqrt{(R_{p}\psi + a_{2})^{2} + b_{2}^{2}}$$
(C19)

Finally, the root envelope coordinates  $\mathbf{x}_{\text{env}}$  and  $\mathbf{y}_{\text{env}}$  may be determined according to equ. (C10).

$$\overline{OS} = x_{c} \tilde{i} + y_{c} \tilde{j} + r_{c} \tilde{n}_{cs}$$
 (C20)

with the help of equ's. (C13), (C14) and (C19), one obtains:

$$x_{env} = -(R_p - b_2) \sin (\tau + \psi) + (R_p \psi + a_2) \cos (\tau + \psi)$$

+ 
$$r_c = \frac{(R_p \psi + a_2) \cos (\tau + \psi) + b_2 \sin (\tau + \psi)}{\sqrt{(R_p \psi + a_2)^2 + b_2^2}}$$
 (C21)

and

$$y_{env} = (R_p - b_2) \cos (\tau + \psi) + (R_p \psi + a_2) \sin (\tau + \psi)$$

+ 
$$r_c = \frac{(R_p \psi + a_2) \sin (\tau + \psi) - b_2 \cos (\tau + \psi)}{\sqrt{(R_p \psi + a_2)^2 + b_2^2}}$$
 (C22)

#### APPENDIX D

#### INITIAL AND FINAL ANGLES FOR TROCHOID AND INVOLUTE GENERATION

When computing gear tooth profile coordinates it is necessary to know the initial and final values of the trochoid generating angle  $\psi$  (see Appendix B) as well as of the involute generating angle  $\alpha$  (called roll angle in Appendix A).

The following shows how the relationship of the trochoidal path of the effective point of the rack cutter with respect to the rest of the gear tooth profile may be utilized to determine these angles for undercut and non-undercut teeth. It must be kept in mind that, while the effective point of the cutter tooth always determines the magnitude of the inner form radius and with that the initial trochoid generating angle  $\psi_{\rm in}$ , it does not furnish the angle associated with the lowest point of the gear root whenever there is a fillet radius on the cutter. This final trochoid angle  $\psi_{\rm fin}$  is defined as that angle for which the effective point of the cutter coincides with the centerline of its trochoidal path (see sections 4 and 5 below).

The initial involute generating angle  $\alpha_{\mbox{in}}$  is a function of the inner form radius. The final involute generating angle  $\alpha_{\mbox{fin}}$  depends on the outside radius of the gear tooth.

Sections 1 and 2, on the trochoid angle  $\tau$  and on the determination of the inner form radius for undercut teeth respectively, are taken with minor nomenclature changes from the

work of R.A. Shaffer: An Analysis of the Undercut Problem in Involute Spur Gearing, Report No.R-1606,OMS5530.11.557, DA Project 505-01-003, Frankford Arsenal, Pitman-Dunn Laboratories, May 1962.

### 1. The Trochoid Angle $\tau$

Figure D-1 shows the trochoid described by the effective point of the indicated rack tooth. The line  $\overline{S'U'}$ , of length  $b_{eff}$ , coincides with the center line  $\overline{SU}$  of the trochoid when the generating angle  $\psi'$ , as used for this purpose, is zero together with the length  $\overline{QS}$ , along the rack pitch line. The trochoid angle  $\tau$  is measured from the trochoid center line and it defines the position of the radius R (used later to get the form radius  $R_f$ ) from the orgin O to the instantaneous location of the effective rack point at U'. Then,

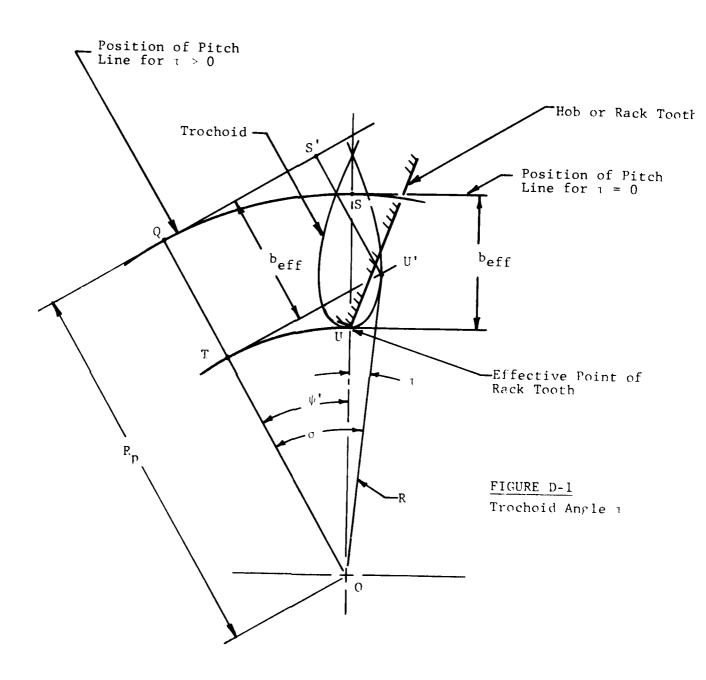
$$\tau = \sigma - \psi' \tag{D1}$$

where

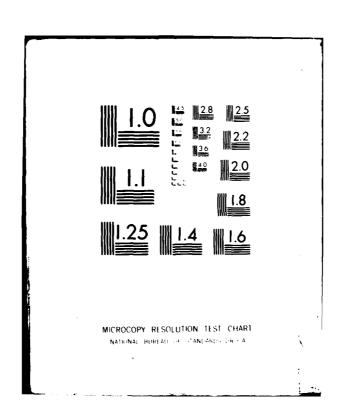
$$\sigma = \ \ TOU' = \tan^{-1} \frac{\overline{TU'}}{\overline{TO}} = \tan^{-1} \frac{\sqrt{R^2 - (R_p - b_{eff})^2}}{R_p - b_{eff}}$$
 (D2)

and

$$\psi' = \frac{\text{arc } \overline{QS}}{R_p} = \frac{\overline{QS'}}{R_p} = \frac{\overline{TU'}}{R_p} = \frac{\sqrt{R^2 - (R_p - b_{eff})^2}}{R_p}$$
(D3)



CITY COLL RESEARCH FOUNDATION NEW YORK AUTOMATED STRENGTH DETERMINATION FOR INVOLUTE FUZE GEARS.(U) AD-A117 593 F/6 19/1 DAAK10-79-C-0251 FEB 82 G G LOWEN. C KAPLAN UNCLASSIFIED ARLCD-CR-81060 NL · 2 ... 3



Substitution of equ's (D2) and (D3) into equ. (D1) furnishes:

$$\tau = \tan^{-1} \frac{\sqrt{R^2 - (R_p - b_{eff})^2}}{\frac{R_p - b_{eff}}{}} - \frac{\sqrt{R^2 - (R_p - b_{eff})^2}}{\frac{R_p}{}}$$
(D4)

## 2. Determination of Inner Form Radius for Undercut Teeth

Figure D-2 shows the side of a basic rack of pressure angle  $\theta$  in line with the base radius  $\overline{OV}=R_{b}$ , of the indicated involute surface. Point V represents the theoretical origin of this involute. The effective point of the rack cutter, shown at point U', is positioned such that the involute profile will be undercut, i.e.  $b_{eff} > R_{p} \sin^{2}\theta$ . The trochoid of this effective point intersects the involute at point P and the length  $\overline{OP}$  represents the inner form radius  $R_{f}$ . Note that the center line of the trochoid is represented by line  $\overline{OU}$ , which forms angle  $\delta$  with the side of the rack tooth.

To obtain an expression for  $\boldsymbol{R}_{\boldsymbol{f}}$  one sets the following angles equal to each other:

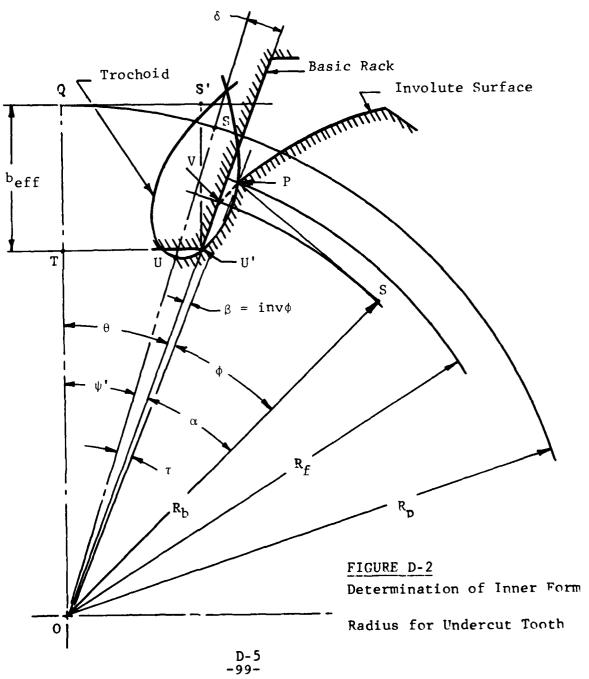
$$\tau = \delta + \beta \tag{D5}$$

where

 $\tau$  = obtained from equ. (D4), with R = R<sub>f</sub>

Further,

$$\delta = \theta - \psi', \qquad (D6)$$



where  $\psi'$  may now be expressed in terms of the rack pressure angle  $\theta$ . [See also equ. (D3)].

$$\psi' = \frac{\widehat{QS}}{R_p} = \frac{QS'}{R_p} = \frac{(R_p - b_{eff}) \tan \theta}{R_p}$$
 (D7)

The angle  $\beta$  has its origin in involutometry. If the angle  $\alpha$  represents the involute generating angle with respect to base radius  $\overline{OV}$ , and if one defines the angle POS =  $\phi$ , then:

$$\beta = INV \phi = tan\phi - \phi = \frac{\sqrt{R_f^2 - R_b^2}}{R_b} - tan^{-1} \frac{\sqrt{R_f^2 - R_b^2}}{R_b}.$$
 (D8)

Substitution of equ's. (D4), (D6), (D7) and (D8) into equ. (D5) furnishes:

$$= \theta - \frac{(R_{p} - b_{eff}) \tan \theta}{R_{p}} + \frac{\sqrt{R_{f}^{2} - R_{b}^{2}}}{R_{b}} - \tan^{-1} \frac{\sqrt{R_{f}^{2} - R_{b}^{2}}}{R_{b}}$$
(D9)

This expression may be solved for  $\mathbf{R}_{\mathbf{f}}$  either by a computer adaptation of the Newton - Raphson method or by a trial and error method.

## 3. Determination of Inner From Radius for Non-undercut Teeth

Figure D-3 shows the effective point of a rack cutter making initial contact with the prospective involute surface above the origin point V of the involute. Since now b  $_{\mbox{eff}}$  <  $R_{\mbox{D}} \sin^2\theta$ , the tooth will not be undercut.

As in Figure B-4 of Appendix B, the contact point Q and with that the effective point lie on the line of action  $\overline{\text{UP}}$ . The inner form radius  $R_f$  may be described as follows:

$$R_f = \sqrt{(QU)^2 + R_b^2}$$
 (D10)

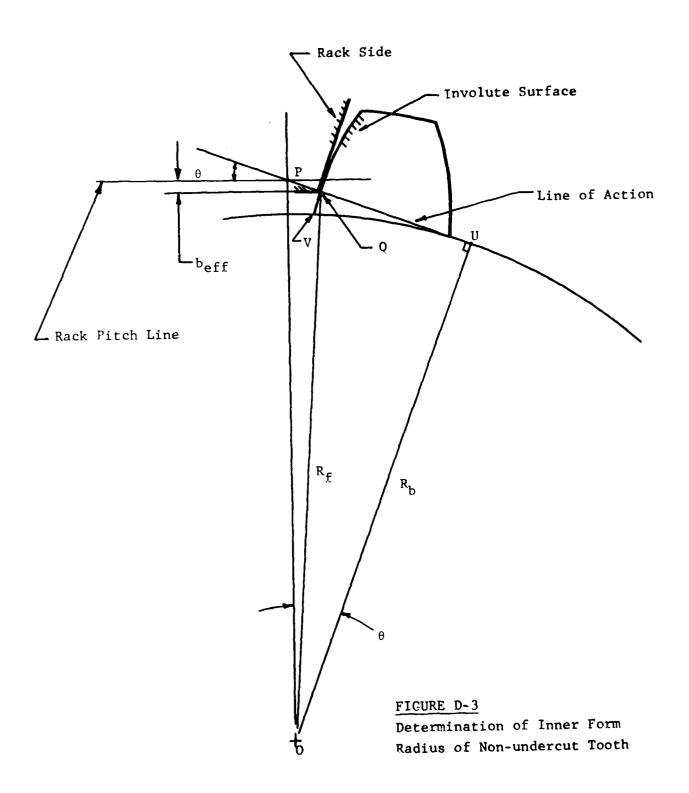
where

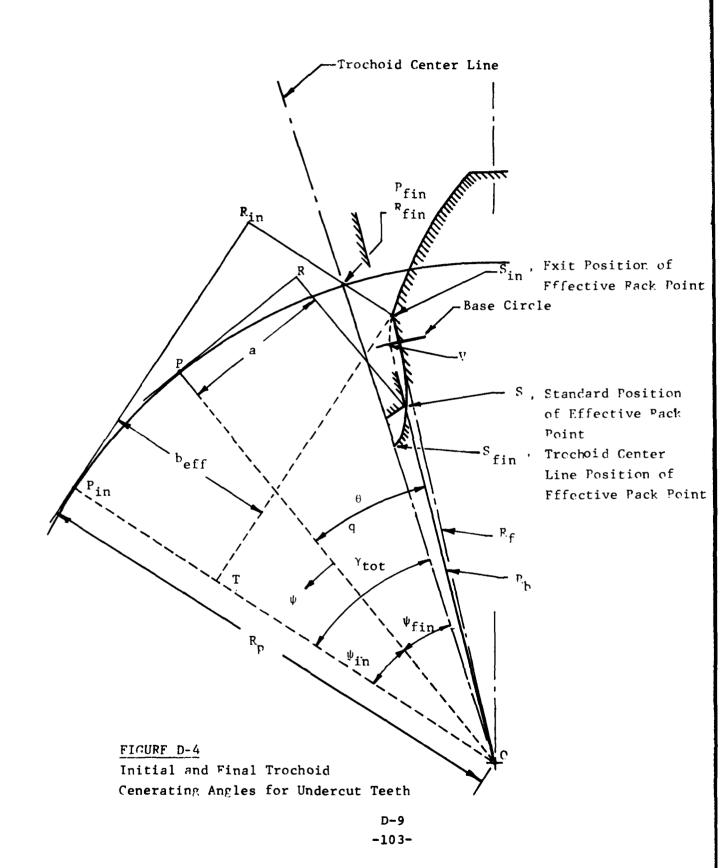
$$\overline{QU} = \overline{PU} - \overline{PQ} = R_{b} \tan \theta - \frac{b_{eff}}{\sin \theta}$$
 (D11)

with the above, equ. (D10) becomes:

$$R_{f} = \sqrt{(R_{b} \tan \theta - \frac{b_{eff}}{\sin \theta})^{2} + R_{b}^{2}}$$
 (D12)

For the special case when  $b_{eff} = R_p \sin^2 \theta$ , equ. (D12) becomes:





 $R_{f} = R_{h} , \qquad (D13)$ 

as is to be expected.

## 4. <u>Petermination of Initial and Final Trochoid Generating Angles</u> for Undercut Teeth

Figure D-4 shows the outline of the rack cutter in its standard position, as defined in Appendix B. The effective point of the rack makes contact with the undercut trochoid, generated by it, at point S. The flank of the rack is in line with base radius  $\overline{OV} = R_b$ , where V represents the origin of the involute curve. The associated line  $\overline{OP} = R_p$  to the pitch point is at the pressure angle 0 of the cutter with respect to line  $\overline{OV}$ . The angle  $\psi$  is zero. Also, the distance  $\overline{PR}$ , along the pitch line and normal to  $b_{eff}$ , equals a, as given by equ. (B2) in Appendix B.

In order for the effective point to make contact with point  $S_{\mbox{fin}}$  at the trochoid center line, the rack pitch line must roll clockwise through the angle  $\psi_{\mbox{fin}}$  until points  $P_{\mbox{fin}}$  and  $R_{\mbox{fin}}$  coincide. The distance rolled through equals  $a=\overline{PR}$ , and therefore:

$$\psi_{\text{fin}} = -\frac{a}{R_{p}} \tag{D14}$$

(Note that the above angle is negative because of the zero position of  $\psi$ .)

When the effective point of the rack cutter is located at point  $S_{in}$ , the intersection of the trochoid and the involute surface,  $\psi = \psi_{in}$ . To determine this angle, one must first find the total roll angle  $\gamma_{tot}$  of the pitch line, associated with the change of the effective point from  $S_{in}$  to  $S_{fin}$ .  $\gamma_{tot}$  depends on the arc length  $\widehat{P_{in}P_{fin}} = \overline{P_{in}R_{in}}$  and it equals the angle between the pitch radii  $\overline{OP}_{in}$  and  $\overline{OP}_{fin}$ . With the inner form radius  $R_f$  known, one obtains

$$\widehat{P_{in}}_{fin} = \overline{TS_{in}} = \sqrt{R_f^2 - (R_p - b_{eff})^2}$$
 (D15)

 $R_{\mbox{\scriptsize f}}$  must be computed with the help of equ. (D9).

Then:

$$\gamma_{\text{tot}} = \frac{\widehat{\text{arc } P_{\text{in}} P_{\text{fin}}}}{R_{\text{p}}} = \frac{\sqrt{R_{\text{f}}^2 - (R_{\text{p}} - b_{\text{eff}})^2}}{R_{\text{p}}}$$
(D16)

Finally, as figure D-4 indicates:

$$\psi_{in} = \gamma_{tot} - \left| \psi_{fin} \right|$$
 (D17)

or

$$\psi_{\text{in}} = \frac{\sqrt{R_{\text{f}}^2 - (R_{\text{p}} - b_{\text{eff}})^2}}{R_{\text{p}}} - \frac{a}{R_{\text{p}}}$$
 (D18)

# Initial and Final Trochoid Generating Angles for Non-Undercut Teeth

Figure D-5 shows the effective point of the rack cutter making initial contact with the involute profile at point S. Since this point lies on the line of action  $\overline{UP}$ , located above the origin V of the involute, there will be no undercutting. (Note that this figure describes the same relationship between rack and involute as Figure B-4 of Appendix B). The distance  $\overline{PR}$  along the rack pitch line equals a, as defined by equ. (B10). The generating angle  $\psi$  equals zero in this initial position, while the angle between the pitch radius  $\overline{OP} = R_p$  (with point 0 not shown) and the base radius  $\overline{OU} = R_p$  equals  $\theta$ , the pressure angle of the rack.

In order for the effective point of the cutter to make contact with point  $S_{\text{fin}}$  at the trochoid center line, the rack pitch line must, as in the last section, roll clockwise through the angle  $\psi_{\text{fin}}$  until points  $P_{\text{fin}}$  and  $R_{\text{fin}}$  concide. The distance rolled through is again given by  $\overline{PR} \approx a$ . (It is important to remember that the length a is now given by equ. (B10), rather than by equ. (B2) of Appendix B). Therefore,

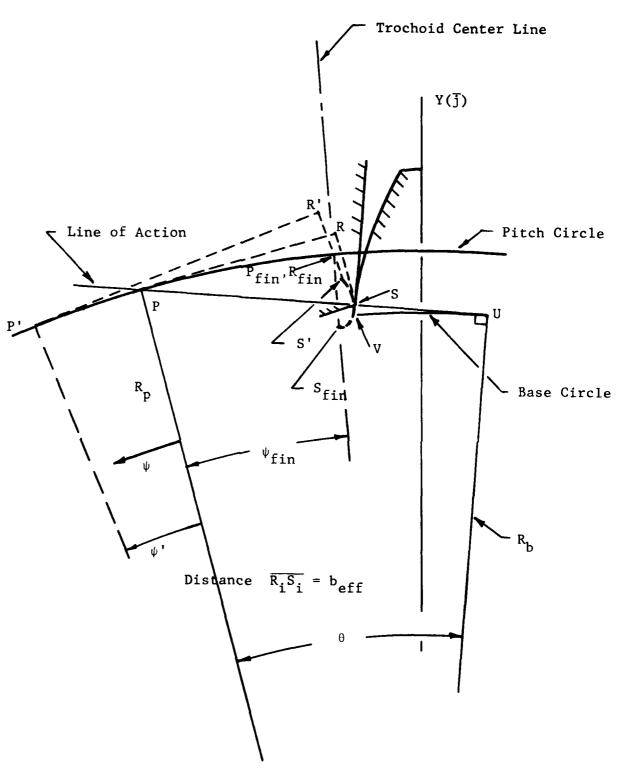


Figure D-5
INITIAL AND FINAL TROCHOID
GENERATING ANGLES FOR NON-UNDERCUT TEETH

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$$\psi_{\text{fin}} = -\frac{a}{R_{\text{p}}} \tag{D19}$$

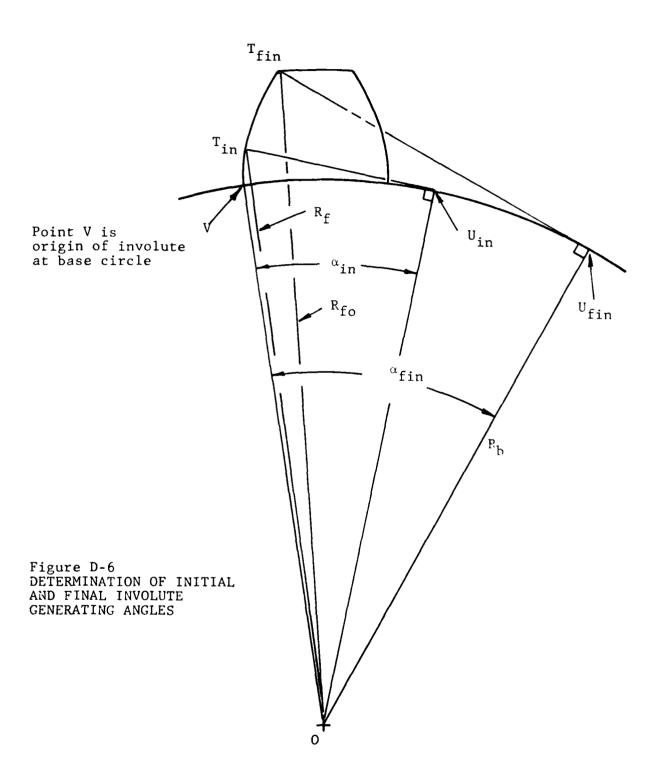
It can be shown that, between points S and  $S_{\text{fin}}$  the trochoid profile moves away from the theoretical involute profile  $\overline{\text{SV}}$ . In addition, point S also respresents the last point of tangency between the trochoid and the involute, as angle  $\psi$  increases. To make this clear, figure D-5 shows that point S', which is associated with the arbitrary positive angle  $\psi$ ', is at a considerable distance from the involute profile. For the above reason

$$\psi_{in} = 0 \qquad (D20)$$

# 6. Initial and Final Involute Generating Angles for Undercut and Non-Undercut Teeth

Figure D-6 indicates that, once the inner form radius is known, the initial involute generating angle  $\alpha_{\mbox{in}}$  may be computed, regardless of whether the tooth is undercut or not, with the help of the following:

$$\alpha_{in} = \frac{\sqrt{V_{in}}}{R_b} = \frac{\overline{T_{in}U_{in}}}{R_b}$$
 (D21)



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Since

$$\overline{T_{in}U_{in}} = \sqrt{R_f^2 - R_b^2}$$
 (D22)

the above becomes:

$$\alpha_{in} = \frac{\sqrt{R_f^2 - R_b^2}}{R_b}$$
 (D23)

Similarly, with the outer form radius  $R_{\mbox{fo}}$  known, the final involute generating angle  $\alpha_{\mbox{fin}}$  is given by :

$$\alpha_{\text{fin}} = \frac{\sqrt{R_{\text{fo}}^2 - R_{\text{b}}^2}}{R_{\text{b}}} \tag{D24}$$

#### APPENDIX E

GEAR STRENGTH CALCULATIONS. DETERMINATION OF LEWIS AND AGMA FACTORS

This appendix first gives a review of the origins of the Lewis gear strength formula and the associated Lewis and AGMA Geometry Factors. + Subsequently, the essentials of a computer determination of the Lewis Factor for any involute gear profile are developed.

For more detail see: AGMA Information Sheet. Geometry Factors for Determining the Strength of the Spur, Helical, Herringborne and Bevel Gear Teeth. AGMA 226.01, American Gear Manuf. Assoc., Aug. 1970.

# 1. Maximum Bending Stress in a Parabolically Shaped Beam

Figure E-l shows a cantilever beam with a parabolic cross section. The parabola originates at point  ${\tt S}$  and is described by

$$y = B x^2, (E1)$$

where B represents a constant. The beam of thickness b experiences the indicated endload  $F_s$ . Using the usual bending stress formula, the maximum bending stress  $\sigma_{max}$  at the abitrary section f-f becomes:

$$\sigma_{\text{max}} = \frac{\text{M c}}{\text{I}} = 1.5 \frac{\text{F}_{\text{s}} \text{ y}}{\text{b } \text{x}^2}$$
 (E2)

where for the present case:

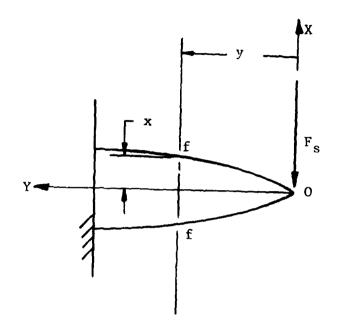
 $M = F_{s}y$  , the moment at section f-f

c = x, the distance to the extreme fiber of the beam

 $I = \frac{b(2x)^3}{12}$ , the moment of inertia of the arbitrary cross section of thickness b.

If one now substitutes the y of equ. (E1) into equ. (E2), the following expression for the maximum bending stress at any point along the beam results:

$$\sigma_{\text{max}} = F_{\text{s}} \frac{1.5 \text{ B}}{\text{b}} \tag{E3}$$



Beam Thickness = b

Figure E-1
Cantilever Beam with Parabolic
Cross Section

The above shows that, the maximum bending stress in a parabolically shaped beam is constant along the beam.

## 2. Derivation of Lewis Gear Strength Formula

on by a contact load F. The line of action of this force passes through point L, the earliest possible contact with a mating tooth, at the end of the involute profile. If there is no load sharing with a second set of mating teeth, i.e. the contact ratio equals unity, this condition also produces the largest possible bending stresses at any cross section below point V, where the line of action intersects the tooth centerline.

The load angle  $\beta$  is defined by the line of action of force F and the normal to the tooth centerline. The bending load  $F_{_{\mbox{S}}}$  is obtained from

$$F_{c} = F \cos \beta, \qquad (E4)$$

and it acts normal to the centerline at point V.

Now, let point V become the origin of a parabola which is tangent to the tooth profile at some section e-e, as indicated in Figure E-2. Since, as has been shown in the previous section, the bending stress along a parabolically shaped beam is constant, section e-e must represent the location of the maximum bending

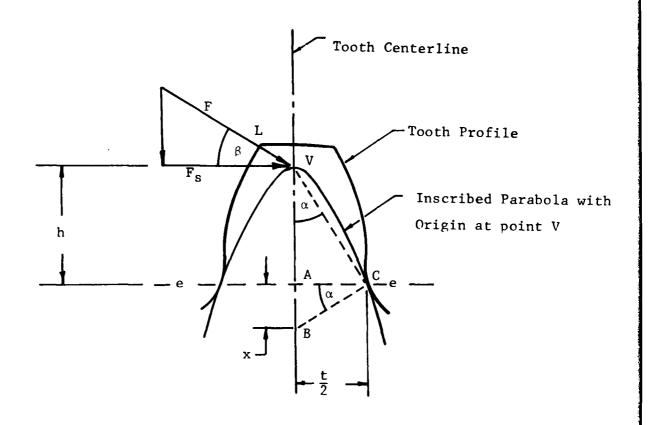


Figure E-2

Background to Lewis Formula

(Line of action of load F, as shown, corresponds to unity contact ratio).

stress of the gear tooth. All other cross sections have portions outside the parabola and will therefore experience lower bending stresses due to the load  ${\bf F}_s$ .

Thus, if one can determine the point of tangency between the parabola and the tooth profile the maximum bending stress may be found with the help of the nomenclature introduced in Figure E-2:

$$\sigma_{\text{max}} = \frac{(F_s h)(\frac{t}{2})}{\frac{b t^3}{12}} = \frac{6 F_s h}{b t^2}$$
 (E5)

where

 $h = \overline{VA}$ , the distance from point V to the tangency section e-e  $t = 2(\overline{AC})$ , the tooth thickness at section e-e

The moment arm h may be expressed in terms of the distance  $x = \overline{AB}$ , which generally is found by graphical means. (See the aforementioned AGMA Standard 226.01). Thus, with the right triangle VCB and the resulting similar triangles AVC and ACB, one obtains the relationship:

$$\frac{\mathbf{x}}{\mathsf{t}/2} = \frac{\mathsf{t}/2}{\mathsf{h}} \tag{E6}$$

and consequently,

$$h = \frac{t^2}{4 x} \tag{E7}$$

Equ. (E7) is now substituted into equ. (E5). This results in the following expression for the maximum bending stress in the gear tooth:

$$\sigma_{\text{max}} = \frac{F_s}{b} \left( \frac{3}{2x} \right) \tag{E8}$$

In order to make equ. (E8) as general as possible, one assumes first that the dimension x was determined for a tooth of arbitrary diametral pitch  $P_d$ . The magnitude  $x_l$  for a tooth of unity diametral pitch will be given by:

$$x_1 = x P_d$$
 (E9)

The maximum bending stress in a tooth of unity diametral pitch then becomes according to the above:

$$\sigma_{\text{max}_1} = \frac{F_s}{b Y} \qquad (E10)$$

where the Lewis factor Y is given by:

$$Y = \frac{2 x_1}{3} = \frac{2 x P_d}{3}$$
 (E11)

This factor may either be directly determined from a unity diametral pitch tooth, or by the indicated multiplication by  $P_{\rm d}$  if x originates from a smaller tooth.

Naturally, for a given load  $\mathbf{F}_{\mathbf{S}}$  and tooth thickness b, the bending stress will be higher for teeth of larger diametral pitch. Therefore, the general form of the Lewis Gear Strength Formula becomes:

$$\sigma_{\text{max}} = \frac{F_s P_d}{b Y}$$
 (E12)

## 3. Determination of Origin V of Lewis Parabola

Before the computer procedure for the determination of the point of tangency between the Lewis parabola and the tooth profile, as well as of the distance x, can be given, it is first necessary to find the distance  $R_v = \overline{OV}$  from the center of the gear to point V, the origin of the Lewis parabola.

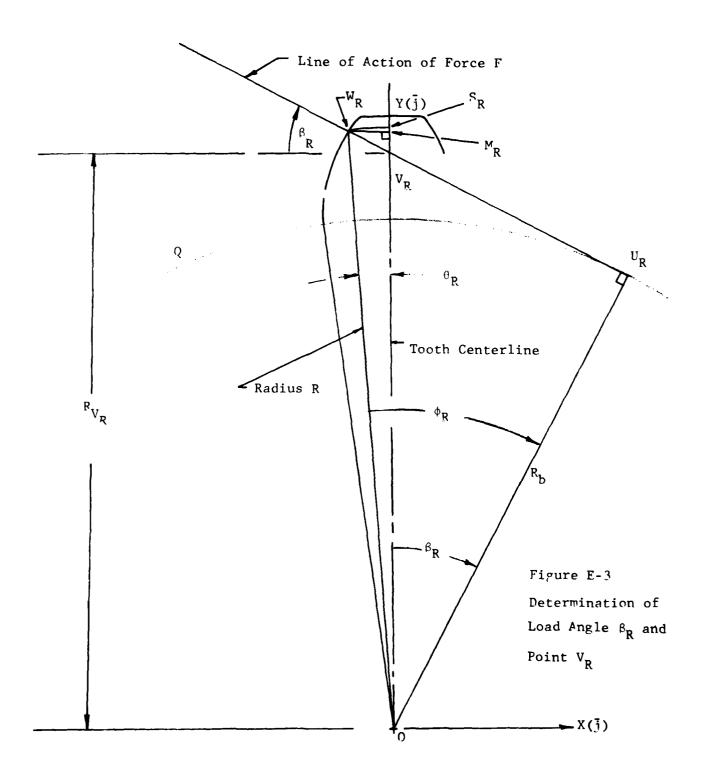
Figure E-3 shows the line of action of force F passing through the arbitrary point  $W_R$  of the involute profile. (Since the force F is always normal to the involute profile, its line of action coincides with line  $\overline{U_RW_R}$ ). The radius  $R=\overline{OW}_R$  is associated with point  $W_R$ .

The load angle  $\beta_R$  of this line of action with the normal to the tooth centerline is identical to the angle between the tooth centerline and the indicated base radius  $\overline{\text{OU}}_R = R_b$ . This later angle may be defined as follows:

$$\beta_{\mathbf{R}} = \phi_{\mathbf{R}} - \theta_{\mathbf{R}} \tag{E13}$$

where

$${}^{\phi}R = \cos^{-1}\frac{R_b}{R} \tag{E14}$$



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and

$$\theta_{R} = \frac{\widetilde{W_{R}}S_{R}}{R}$$
 (E15)

The above arc length is equal to one half of the circular tooth thickness  $\mathbf{T}_R$  at radius R. To determine this tooth thickness, one makes use of the following relationship from involutometry which allows the determination of a circular tooth thickness  $\mathbf{T}_2$  at a radius  $\mathbf{R}_2$  if the tooth thickness  $\mathbf{T}_1$  at radius  $\mathbf{R}_1$  is known:

$$T_2 = T_1 - \frac{R_2}{R_1} - 2R_2 (INV \phi_2 - INV \phi_1)$$
 (E16)

where

$$INV \phi_{i} = tan \phi_{i} - \phi_{i}$$
 (E17)

To adapt the above to the present case, it is necessary to know the actual circular tooth thickness  $T_{cs}$  at the standard pitch radius  $R_p$  of the gear. (The magnitude of  $T_{cs}$  depends on whether the gear is standard or modified). The associated angle  $\phi$  corresponds always to the pressure angle  $\theta$  of the rack cutter.

Equ. (E15) for  $\theta_{\mbox{\scriptsize R}}$  may now be expressed with the help of equ. (E16) in the following manner:

$$\theta_{R} = \frac{T_{R}}{2R} = \frac{1}{2R} \left[ T_{cs} \frac{R}{R_{p}} - 2R \left( INV \phi_{R} - INV \theta \right) \right]$$
 (E18)

or

$$\theta_{R} = \frac{T_{CS}}{2R_{D}} - INV \phi_{R} + INV \theta$$
 (E19)

Finally, Equ. (E13) for load angle  $\beta_{\mbox{\scriptsize R}}$  becomes:

$$\beta_{R} = \cos^{-1}(\frac{R_{b}}{R}) - \frac{T_{cs}}{2R_{D}} + INV \phi_{R} - INV \theta$$
 (E20)

For the case when the contact ratio of the mesh is unity and therefore the crucial contact is made at the end of the involute, one lets  $R=R_{_{\hbox{\scriptsize O}}}$ , the outside radius of the gear, in equ. (E20). This gives the load angle  $\beta$  for the Lewis parabola:

$$\beta = \cos^{-1}\left(\frac{R_b}{R_o}\right) - \frac{T_{cs}}{2R_p} + INV \phi_{R_o} - INV \theta$$
 (E21)

where

$$\phi_{R_o} = \cos^{-1}(\frac{R_b}{R_o})$$
 (E22)

To determine the position of point  $V_{\rm R}$  , where the line of action intersects the tooth centerline, one considers the following: (See Figure E-3)

$$R_{V_R} = \overline{OV}_R = \overline{OM}_R - \overline{M}_R \overline{V}_R$$
 (E23)

Since the line  $\overline{\boldsymbol{W}_{R}\boldsymbol{M}_{R}}$  is normal to the tooth centerline,

$$\overline{W_R^M}_R = R \sin\theta_R$$
, (E24)

while

$$\overline{OM}_{R} = R\cos\theta_{R}$$
 (E25)

and

$$\overline{M_R V_R} = R \sin \theta_R \cdot \tan \beta_R$$
 (E26)

Substitution of equ's. (E25) and (E26) into equ. (E23) leads to the position of point  $\mathbf{V}_{R}$  with respect to center 0 of the gear:

$$R_{V_R} = R \left( \cos \theta_R - \sin \theta_R \tan \theta_R \right)$$
 (E27)

For the case when  $R = R_0$ , the above expression becomes:

$$R_{v} = R_{o}(\cos\theta_{R_{o}} - \sin\theta_{R_{o}} \tan\beta) , \qquad (E28)$$

where  $\beta$  is given by equ. (E21) and  $\theta_{\mbox{$R$}_{\mbox{$O$}}}$  is obtained with the help of equ. (E19) , i.e.:

$$\theta_{R_O} = \frac{T_{CS}}{2R_D} - INV \phi_{R_O} + INV \theta \qquad (E29)$$

again, 
$$\phi_{R_0} = \cos^{-1} \frac{R_b}{R_0}$$
 (E30)

## 4. Computer Determination of Lewis and AGMA Geometry Factors

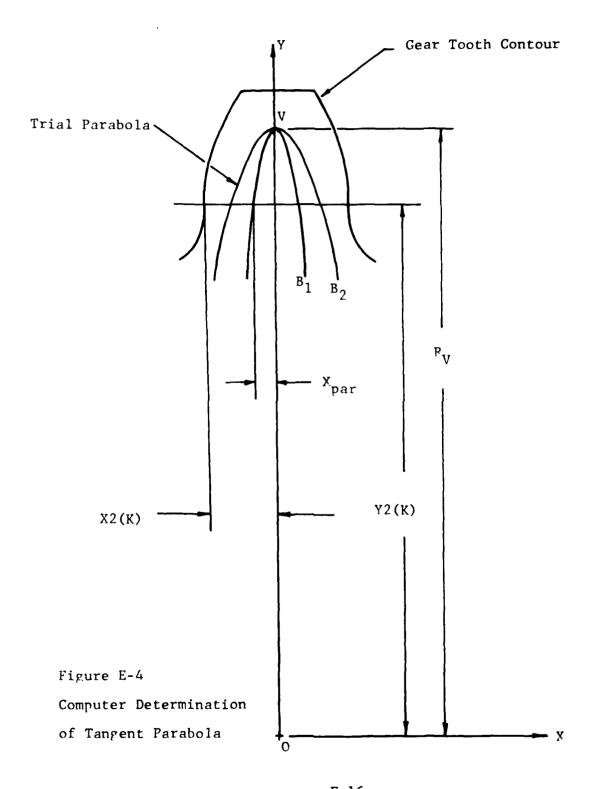
The computer determination of the Lewis and the AGMA Geometry factors requires that the coordinates of the gear tooth profile are fully known and stored in the associated vectors X2(K) and Y2(K). Subsequent to this, the following operations are performed (See Figure E-4):

- a. The distance  $R_V = \overline{OV}$ , of the origin V of the Lewis parabola, is found with the help of equ. (E28).
- b. To accommodate the specific situation, the equation of this parabola becomes:

$$R_{v} - Y2(K) = B X_{par}^{2}$$
, (E31)

where  $X_{par}$  is the x-coordinate of the parabola, associated with the coordinate Y2(K). Again, as in equ. (E1), B represents the constant of a specific parabola. Since  $X_{par}$  is required, equ. (E31) becomes:

$$X_{par} = \sqrt{\frac{R_{v} - Y2(K)}{B}}$$
 (E32)



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- c. In order to find the tangent parabola and with that the value of the parameter x of the Lewis formula, the appropriate constant B must be determined. To this end, a comparatively large B is first chosen to make sure that the associated parabola does not contact the tooth profile. The inner parabola of Figure E-4, with the constant B<sub>1</sub>, represents such a case. As the magnitude of B is progressively decreased, the resulting parabolae approach the tooth profile more closely. (The constant B<sub>2</sub> of the outer parabola of Figure E-4 is smaller than the constant B<sub>1</sub> of the inner one). Eventually, the tangent parabola will have the correct value of B.
- d. The above process is accomplished by computing for a given value of B all  $X_{par}$  as functions of the full range of the Y2(K). Each  $X_{par}$  is compared with the corresponding value of X2(K) of the tooth profile. Whenever

$$| X2(K)| - | X_{par} | \leq 0$$
, (E33)

the correct value of B was used and the associated values of  $X2(K_{tan})$  and  $Y2(K_{tan})$  lie on the tangency

cross section of the gear tooth. The tooth thickness t as defined by Figure E-2 then becomes:

t = 2 [| 
$$X2(K_{tan})$$
 |] (E34)

e. Finally, the Lewis distance x of equ. (E6) may be found by:

$$x = \frac{[t/2]^2}{h} = \frac{[X2(K_{tan})]^2}{R_{v} - Y2(K_{tan})}$$
(E35)

The AGMA Geometry factor  $\mathbf{Y}_{\mathbf{AGMA}}$  is obtained from the substitution of the above into

$$Y_{AGMA} = \frac{1}{\frac{\cos \theta}{\cos \theta} \left(\frac{1.5}{x} - \frac{\tan \theta}{2[|X2(K_{tan})|]}\right)}$$

(E36)

#### APPENDIX F

#### PROGRAM MATRIX

The present appendix describes and lists program MATRIX which is used for the determination of the sizes of the various arrays in programs LEWIS CIRCLES and LEWIS ENVELOPE. A sample of the program, together with two output pages is given in section 5 below.

1. Determination of Length of Vectors X1 and Y1
(Used in programs LEWIS CIRCLES and LEWIS ENVFLOPE.)

The vectors X1 and Y1 are used for the initial storage of the involute coordinates of the tooth profile. The common length of these vectors is determined with help of equ. (1) i.e.

Y1-rows = 
$$\frac{\alpha \sin^{-\alpha} \sin}{\Delta \alpha}$$
 (F1)

The explanation for the above nomenclature follows equ. (1) in section 2.

2. Determination of Sizes of Vector Y and Array X (Used mostly in program LEWIS CIRCLES.)

The number of rows, both of the vector Y and the array  $\chi$ , depends on the number of one one-thousandth of an inch contained in the vertical distance between the initial point of the first fillet

circle and the final point of the last one.

The y-coordinate  $y_{in}$  of the first point of the first circle is that of its effective point. It is determined for  $\psi = \psi_{in}$  according to equ. (B9) or equ. (B25):

$$y_{in} = (R_p - b_{eff})\cos(\tau + \psi_{in}) + (R_p\psi_{in} + a)\sin(\tau + \psi_{in})$$
 (F2)

where, for an undercut tooth

$$\tau = n = \frac{\theta}{2} + \theta$$
, according to equ. (B9) (F3)

 $\psi_{in}$  = initial roll angle of rack pitch line according to equ. (D18)

For a non-undercut tooth:

$$\tau = \eta = \frac{\theta_b}{2} + \theta + \frac{b_{eff}}{R_p \sin \theta} - \tan \theta$$
, according to equ.(B25) (F4)

 $\psi_{in}$  = 0, according to equ. (D20)

The final point  $y_{fin}$  of the last circle is obtained according to equ. (3) or (5), i.e.:

$$y_{fin} = y_{cfin} - r_c$$
 (F5)

 $y_{cfin}$  represents the center of the fillet circle for  $\psi = \psi_{fin}$ . Then, according to equ's.(C4) or (C6):

$$y_{efin} = (R_p - b_2)\cos(\tau + \psi_{fin}) + (R_p\psi_{fin} + a_2)\sin(\tau + \psi_{fin})$$
(F6)

When the tooth is undercut the angle  $\tau$  is used according to equ. (F3), otherwise equ. (F4) must be used. The angle  $\psi_{\mbox{fin}}$  for undercut and non-undercut teeth is the same. It is given by equ. (D14) or equ. (D19).

Thus, the number of rows of X and Y are found from:

$$y-rows = 1000(y_{in} - y_{fin})$$
 [in form of an integer] (F7)

The number of columns of the array X depends on the number of circles involved. This in turn is a function of the angular increment  $\Delta \psi$  as well as  $\psi_{\mbox{in}}$  and  $\psi_{\mbox{fin}}$ . Therefore, as in equ. (10):

X-columns = 
$$\frac{\psi_{\text{in}} - \psi_{\text{fin}}}{\Delta \psi}$$
 [also made into an integer] (F8)

3. Hand Computation of Sizes of Vectors X2 and Y2
(Used in programs LEWIS CIRCLES and LEWIS ENVELOPE)

The size of vectors X2 and Y2, which contain the final tooth profile coordinates, are not computed in program MATRIX, but may be obtained by adding the results of equ's. (F1) and (F7):

Total rows of X2 and Y2 = Y1-rows + Y-rows (F9)

The above will give an exact result for use in program LEWIS CIRCLES and a somewhat larger than needed number for program LEWIS ENVELOPE (The y-coordinates of the root are not generally computed every one one-thousandth of an inch by this routine.)

4. Description of Computer Program MATPIX (See section 5 of this Appendix)

## a. Input Parameters

The following input data are required:

N = number of teeth in gear under consideration

BEFF =  $b_{eff}$  for unity diametral pitch (See Appendix B)

PD =  $P_d = 1.000$ , the diametral pitch

THETAD =  $\theta$ , the pressure angle of the hob

 $CAPTCS = T_{CS}$ , the circular tooth thickness of the gear at

the standard pitch radius  $R_p = \frac{N}{2P_d}$ , in terms of  $P_d = 1.000$ 

RC =  $r_c$ , the rack fillet radius for unity diametral pitch

KADD = Addendum constant of gear, with the magnitude of the addendum defined by KADD/PD, in terms of  $P_{\mbox{d}}$  = 1.000

DELPSI =  $\Delta \psi$ , the increment of the roll angle of the rack pitchline on the pitch circle of the gear

DELAL =  $\Delta\alpha$ , the increment of the involute generating angle

### b. Computations

The initial computational sequence in program MATRIX is in many ways identical to that of the early parts of programs LEWIS (See section 2). Following the test which establishes whether the tooth under consideration is undercut or not, the appropriate parameters for equ's.(F1) to (F8) are computed.

YIN =  $y_{in}$ , according to equ. (F2)

YCFIN =  $y_{cfin}$ , according to equ. (F6)

YFIN =  $y_{fin}$ , according to equ. (F5)

The integral number of rows of X and Y is found in lines 64 to 66, according to equ. (F7). The integral number of columns of array X is determined according to equ. (F8) in lines 67 an 68. The number of rows of vectors X1 and Y1, based on equ. (F1), are found in a similar manner in lines 69 and 70.

### c. Output Paramerers

(All lengths are given in inches and all angles are given in degrees)

The output of the program first states the numerical values of the input parameters N, BEFF, PD, THETAD, CAPTCS, RC, KADD, DELPSI, and DELAL.

Subsequently, it is noted whether the tooth is undercut or not. In addition, the numerical values of the following computed quantities are given:

 $RF = R_{f}$ 

 $ALPHIN = \alpha_{in}$ 

 $ALPHFIN = \alpha_{fin}$ 

 $PSIIN = \psi_{in}$ 

PSIFIN =  $\psi_{fin}$ 

 $YIN = y_{in}$ 

 $YCFIN = y_{cfin}$ 

 $YFIN = y_{fin}$ 

Finally, the number of rows of the vectors X1 and Y1, together with the size of the tooth root array X, are printed out.

# 5. Sample Program and Sample Output

The following pages contain a listing of program MATRIX as well as two sample outputs.

The programs were run for the same teeth as found in programs LEWIS CIRCLES and LEWIS ENVELOPE in section 3 of this report.

The input data for the first run on p. F-12 are as follows:

N = 36,

BEFF = 1.215 in.,

PD = 1.000,

THETAD =  $20^{\circ}$ ,

CAPTCS = 1.8460 in.,

RC = 0.040 in.,

KADD = 1.425 in.,

DELPSI = 0.125 degrees

DELAL = 0.25 degrees

The program found that the tooth is not undercut. The various output parameters from RV toYFIN may be used for checking. Finally the array sizes are given as:

Length of Vectors Xl and Yl : 94

Length of Vector Y : 370

Size of Array X :  $370 \times 85$ 

The lengths of vectors X2 and Y2 are obtained by addition of the lengths of Y1 and Y, according to equ. (F9), i.e.,

94 + 370 = 464

The second run of the program on page F-13 concerns itself with an undercut 12 tooth pinion.

```
$JCR
     C
     C
              * # # # WATRIX
                             * * * * *
     C
     C
     C
           IMPLICIT REAL #9(4-H,C-Z)
 2
           COMMON REPREPATHETA
           REAL N.KADD
           READ (5,51) N. BEFF, PD, THETAD, CAPTOS, RC
           READ(5,52) KACD
           REAC(5.53) DELPSI.DELAL
        51 FORMATIEF9.51
 7
        52 FORMAT(F9.5)
 8
        53 FORMAT(2F9.5)
9
           WRITE (6.1) N.BEFF.PD
10
         1 FCRMAT("1"," = ",F9.5,5X,"BEFF = ",F9.5,5X,"PE = ",F9.5)
1.1
           SRITE(6,3) THETAD, CAPTOS, RC
12
13
         3 FCRMAT(//' THETAD = +.F9.5,5X, +CAPTCS = +.F9.5,5X, +RC = +.F9.5)
           WRITE(6.2) KACD.DELPSI.DELAL
: 4
1 6
         2 FORMAT(//! ','KADO = ',F9.5,5X,'DELPSI = ',F9.5,5X,'DELAL = ',F9.5//)
16
           PI = 3.14159
           7 = PI/18C.
17
18
           THETA = THETAC+Z
           P = N/(2.*PC)
19
           RB = RP#CCCS(THETA)
. C
21
           RO = RP+KACC/PC
>2
           B = REFF/20
23
           THETAB = CAPTCS*DCDS(THETA)/RB+2.*(DTAN(THETA)-THETA)
24
           EPS = THETAB/2.00 + THETA
25
           CELTA = P/(RP+CSIN(THETA)) - DTAN(THETA)
26
           PALLOW = RP*DSIN(THETA)*DSIN(THETA)
21
           IF(BEFF.GT.BALLOW) GC TO 41
28
           IF (BEFF.LE. BALLOW) GC TO 42
              C
     С
                      TECTH IS UNDERCUT #####
              ****
              **+****
25
        41 \Delta = (RP-R)*CTAN(THETA)
30
           TAL = EPS
           PT = RC
31
           C = 46
32
33
           RF=DEKKER(8T,C,1.0-16,1.0-18,100)
           ALPHIN = CSCRT(DARS(RF#RF-RP#RB))/RB
34
35
           ALPHF = DTAN(DARCOS(RR/RC))
           GAMT = (CSCRT(DAHS(RF#RF-(RP-BEFF)##2)))/RP
36
37
           PSTIN = -A/RP + GAMI
           PSIFIN = -A/4P
3 8
35
           WRITE(6,65)
        65 FORMATT// *****
4 C
                               TCOTH IS UNDERCUT
           GO TO 43
41
     C
     C
     C
                      TOOTH IS NOT UNDERCUT *****
              ***
              **********
        42 \Delta = P/CTAN(THETA)
42
           TAU = EPS + CELTA
43
44
           RF = DSGRT(DABS((RB*DTAN(THETA).-B/DSIN(THETA))**2+R6*RB))
45
           \DeltaLPHIN = DSQRT(DABS(RF*RF-RE*RB))/RB
46
           IF (alphin.le.o.col) alphin = \sigma
                                     F-9
                                    -137-
```

```
47
           SUPPR = CTAN(CARCCS(RB/RC))
4 P
           FSIIN = C.C
45
           PSIFIY = -1/20
50
           WRITE (6,66)
66 FCRMAT(// #####
                             TOOTH IS NOT UNDERCUT
                                                     ******//)
5 3
        43 CONTINUE
53
          WRITE(6,62) RF
       62 FURMAT(//! *, *RF = *, F9.5//)
54
    С
    Ç
              C.
              ##### FOR ALL TYPES OF TEETH
              ë ë
           FSIINU = PSIIN/Z
د د
           PSIFC = PSIFIN/Z
5.7
           ALINE = GLPHIN/?
5 &
           TEFIND = ALPHF/7
    С
55
           92 = B - RC*DSIN(THETA)
60
           \Delta 2 = \Delta - RC * CCCS (THETA)
           YIN = (RP-R) * CCCS(TAU + PSIIN) + (RP*PSIIN + A) * USIN(TAU + PSIIN)
t 1
15
           YCFIN = (RP-82)*DCCS(TAU+PSIFIN)+(RP*PSIFIN+82)*PSIN(TAU+PSIFIN)
63
           YFIN = YCFIN - RC
           RCW = YIN - YEIN
64
65
           PCh = RCh + 1CCC
66
           IRCh = RCh
67
           COL = (PSIIND - PSIFD)/DELPSI
.. ε
           100L = 00L
10
           RUNIAV = (ALPHE-ALPHIN)/(Z#CELAL)
70
           IRINV = MCWINV
71
           WRITE(6.5) ALIND.ALFIND
       5 FORMAT(//* *, "ALPHIN = *, F9.5, 5X, *ALPHFIN = *, F9.5)
72
7.3
           WRITELE, 86) PSIIND, PSIFD
74
       86 FORMAT(// '. PSIIN =', F9.5, 5X, 'PSIFIN =', F9.5)
           WRITE(6,92) YIN.YCFIN.YFIN
75
76
           WRITE (6,85) IRINV
          FORMAT(///// 1,5x, LENGTH OF ROWS FOR INVOLUTE VECTORS X1 AND Y1
11
          1.//* RCWS =*,16)
7 8
           WKITE(6,87)
          FORMAT(// 1,5%, NUMBER OF ROWS AND COLUMNS FOR TOOTH ROOT ARRAYS!
       A 7
73
          11
80
           WRITE(6,83) [RCW, [CCL
          FCRMAT(//*GY[N=*,F11.5,5X,*YCFIN=*,F11.5,5X,*YFIN=*,F11.5) .
13
       82
          FCRMAT(// *, * RCW=*, 16,5X, * COL=*, 16)
۲2
<u>د</u> نو
           STCP
+4
           END
PE
           COUBLE PRECISION FUNCTION DEKKER (B.C. EPS, TCL. LIMIT)
     C
     C TITLE- CEKKER'S ALGCRITHM
     C PURPOSS- TO FIND A ROOT OF A NON-LINEAR ALGEBRAIC EQUATION F(X)=0.
     C DESCRIPTION OF PARAMETERS
     C
           P = THE LATEST ITERATE AND CLOSEST APPROXIMATION TO THE ROOT.
     C
           A = THE PREVIOUS ITERATE.
           C = THE PREVIOUS CR AN OLDER ITERATE.
     C
     C COMMENTS
           AT ALL TIMES 8 AND C BRACKET THE ROOT.
     C
           INITIALLY A IS SET EQUAL TO C.
     C
                                       F-10
```

-138-

```
USE EPS=1.0-7 AND TCL=1.0-10.
        SUPRCUTINES AND/OR FUNCTION SUBPROGRAMS REQUIRED- F(X)
 € €
             IMPLICIT REAL+8 (A-H,O-Z)
 17
             0 = C
 99
             KCUNT =- 1
 45
             FCA=F(A)
 50
             FOB=F(E)
 91
             FCC=F(C)
 52
             IF((FCE*FCC).GT.O.DO)GO TO 6
 93
           8 KOLNT=KOLNT+1
 94
             IF(DARS(FCR).LE.DABS(FCC))GC TO 7
 5,5
             T=P
 56
             6 = C
 97
             CEKKER=B
 98.
             C = T
 55
             A = C
100
             T=FC8
1CF
             FOR=FOC
1f2
             FOC=1
103
             FOA=FCC
104
           7 IFIFCP.EC.O.DO)RETURN
105
             TOL1=2.DO#EPS#CABS(B)+0.500#TCL
116
             IF(DADS(E-C).LE.2.DC*TCL1)RETURN
107
             IF (DAPS(FER-FCA).LT.1.C-70) RETURN
108
             IF (KEUNT.GE.LIMIT)GC TO 12
105
             \Gamma I = A - \{(B - A) + FCA\}/\{FCB - FOA\}
110
             PP=CI
             IF(.NOT.(((C.LE.DI).ND.(DI.LE.B)).OR.((B.LE.DI).AND.(DI.LE.C))))
111
            1 PP=(0+C1/2.DC
112
             IF((((C.LE.DI).AND.(DI.LE.B)).OR.((B.LE.DI).AND.(DI.LE.C)))
            1 .ANC.(DARS(CI-R).LT.TCL1)) PP=B+DSIGN(TOL1.(C-B))
113
             IF(CARS(CI-C).LT.CARS(R-C)/4.00)RP=(B+C)/2.00
114
             \Delta = P
115
             FC4=FCB
             3 = 6 b
116
             CEKKER=8
117
             FOR = F(F)
116
             IF((FCE*FCC).GT.O.DO)C=A
115
120
             IF((FOE*FCC).GT.O.DO)FOC=FOA
121
             9 JT 00
122
           6 PRITE(6,15)
122
             STOP
124
          12 WRITE (6.13)
175
             STCP
          15 FURRATION FOR THE INPUT CSTA. F(B) HAS THE SAME SIGN AST.
126
            ! * F(C)*)
127
          13 FORMATILE-1. NUMBER OF ITERATIONS HAS EXCREDED LIMIT. STOP COMPUTA
            111CNS. 1)
178
             ENC
             DOUBLE PRECISION FUNCTION FIX)
1:5
             IMPLICIT RESLER (A-H, 0-Z)
130
             COMMON REPREPATHETA
121
122
             F=CATAN(CSCRT(X**2-(RP-R )**2)/(RP-R ))-DSCRT(X**2-(RP-R )**2)/RP
            1-THETA+(HP-8 )*CTAN(THETA)/RP-DSQRT(X**2-R8**2)/R9*DATAN(DSQRT(X
            2**2-3L**5)/86)
122
             RETURN
                                         F-11
174
             ENC
                                        -139-
```

N = 16.CCCC0 PEFF = 1.215GG PD = 1.00000

THETAE = 20.00000 CAPTCS = 1.84500 RC = 0.04000

KACC \* 1.42500 CELPSI = 0.12500 CELAL = 0.25000

\*\*\*\*\* TCCTH 15 NCT LNDERCLT \*\*\*\*\*

RF = 17.11373

ALPHIN = 8.82052 ALPHFIN = 32.35465

PSIIN = 0.00000 PSIFIN =-10.62579

VIN 17.C7761 YCFIN= 16.74754 YFIN= 16.70754

LENGTH OF PONS FOR INVOLUTE VECTORS XI AND YI

ROLS = 94

NUMBER OF RONS AND COLUMNS FOR TOOTH ROOT ARRAYS

RCh\* 370 CCL\* A5

39912 RYTES 48 RYTES, TOTAL AREA AVAILABLE= 8157 RYTES, AKRAY AREA= CRUECT CCCE= CORE LSAGE

O, NUMBER OF EXTENSIONS= O. NUMBER OF WARRINGS= AUPBER CF FRRCRS= CTAGACSTLCS

FRIDAY 15 34.35 0.05 SEC. hATFIV - JUL 1973 VIL4 C.63 SEC. EXECUTION TIME COMFILE TIME=

15 MAY AL

15 MAY RI

FRIDAY

17.27.08

48 BYTES, TOTAL AREA AVAILABLE 39912 BYTES

C.63 SFC. EXECUTION TIME=

#3611 311403

CIAC.CSTICS CORE LSASE

O. NUMBER OF WARNINGS ALMIER OF ERRORS=

CHJECT COCE #152 BYTES, ARRAY ARF4=

25

בנו:

RCH- 395

0.08 SEC. NOTFIV - JUL 1973 VIL4

O. NUMBER OF EXTENSIONS=

LENCTE OF RCHS FCR INVCLUTE VECTORS X1 AND Y1

AUMPER OF PENS AND COLUMNS FOR TOOTH PECT ARRAYS

F-13 -141-/-/42-

CAPTCS = 1.57079 THE TAL = 20.00000 KAEE = 1.COCCC

CFLAL = 0.25000 DELPSI = 0.5CC00

RC = 0.30000

Pn = 1.00000

PEFF = 1.05260

13°CCCC

\*\*\*\* TECTH IS UNDERCLT \*\*\*\*

RF = 5.65550

ALPHFIN = 42.15910 ALPHIN = 4.49751

F91FIN =-17.19548 PSIII. = 8.97016

YCF LN= 4.89998 5.59562

4.59998

YFIN=

¥ [ R =

## APPENDIX G

## PROGRAM LEWIS CIRCLES

```
1000 301d
 16/19/04
                                                                                                                                                                                                                                                                                                                                                                                                                                                                              FCRMAII//**, *KADD =*,F9.5,//) .
hF1]E(6,4) Delal,CELPSI,CE(CCN,CCNB
FCRM41(//**,*)CELAL =*,F9.5,5x,*CELPSI =*,F9.5,5x,*CELCCN= *,F5.5,
                                                                                                                                                                                                                                                                                                                                                                                                                 hrife(6,1)A, UEFF,PC

| FLAMAT(11,1,1 = ",F5.5,5x," EEFF = ",F5.5,5x,"PC = ",F9.5)

hrite(6,2) ine tac, Capics,RC

3 FCRMAT(//" THETAC = ",F5.5,5x,"CAPICS = ",F9.5,5x,"RC = ",F9.5]
                                                                                    SEARCH FUR CENE (2) THAT CIVES LEWIS PAFTEULD WHICH IS TANCENT TO TECTH PROFILE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        1867ai = CPFCstLCLS(1813)/RG + 2.*fLl7N(18(3)-18E1A)
LPs = FHL18672-EC + 1811A
18(70) = TFF18672
   LFTL = 81135
                                                                                                                                                                      EIMENSION XILIDO) + YILIOO)
UIMENSION YELUO XXIACC, IOU) + X2150 (1, Y215CC)
INTEGER BEUIN, FINISH
                                                                                                                                                                                                                                      REAC(5,517)
10 FCRPAT(11)
99 REAL(5,51) N. ALFF, FD. THE 12G, CAP 1CS, RC
REAC(5,52) KACD
FEAC(5,53) DELPSI, CE LAL
REAC(5,53) DELCCA, CCNP
REAC(5,55) NR 11F
                                                                                                                                                                                                                 REAL KCOMFONIACONINATION
    71 4 M
                                                                                                                                                 COMMENT REALMETEN, C-2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             INITIALIZE METRIX
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     HE = FFACCES(THETA)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  CC 77 1=1,400
CC 76 J=1,90
CC 16 J=1,90
XX LC 11,13 = -1C, CC C
78 LC NT 1N LE
                                                                                                                                                                                                                                                                                                                                  1SICP = 1SICP - 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     13x, "CLNB = ", F9.51
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            THILE = THETAU+Z
                                                        LEN IS LIMILES
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          1304.5)/7 = 31
                                                                                                                                                                                                                                                                                                                                                51 FCRMAT(6F9.5)
52 FCRMAT(F9.5)
53 FLHMIT(2F9.5)
54 FCRMIT(2F5.5)
55 FCHMAT(11)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  65151° E = 1d
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               d = 1,011/P)
                                                                                                                                                                                                                                REDL KAUC
      .,
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      FORTHAR IN G LEVEL
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      ب ب ر د ن
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  0040
0041
0047
0043
```

```
#FILE (6,71) TELTOD ( AMID : L FSD + DELITC : TAUC
TL + ( PMAIL / ' THE TAE = ' , +9 . 2 , 5 x + 'GAPT = ', +5 x , 5 ; +2 ; +2 ; +2 ; +5 x , +7 5 x
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 73 FLRMAIL " RF = ", FG. 5, 5x, " RE = ", FG. 5, 5x, " HO = ", FG. 5, 5 }" FF = ", FG. 5/)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  KF = DSGRT(DABS (IR E*E JAN (IF ETA)-B/DS IN (THE TA)) + *2+KE*R E))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   1 //. * * * * * *
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             (//....
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   FF=LEKKEK[ET, C, 1, D-16, 1, C-18, 100)
.AIFFIN = D.SCHI(CABS(KF*RF-KE*RE))/Rd
.IPFF = GTANIDAFCCS(FE/FC))
.CAMI = (CS_RILLABS(RF*FF-(FP-BEFF)**2))/RF
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      coccananoporerantescentrations and content and content
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             FCF4AT(//* **** TCCIH IS NOT UNTERCUT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            CELTA = B/IRB*DSINITETAT) - DIANITETA)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   GAPT = 0.0
ALPLIN = CSARILCABSIFF*FF-FF*RE)!/RE
IF (ALPLIN-LE-0.001) ALPHIN = C
ALPHF = CIANICAKLUSIRE/F()!
FSIIN = C.C
PSIF = -4/FP
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        ICCIP IS UNCERCLE
                                      EALLUM = RP#USINITHEIA)*DSINITHETA)
IFIBEFF.GI.EALLUM) GC IC 41
IFIBEFF.LE.BALLUM) GC IC 42
                                                                                                                                                                                                                                                                                      ***** TOUTH IS ONDERCY *****
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     MEITE (6,7C) RP, FL, HC, PF
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             32 = E - KC#USIN(1FL1A)
42 - A - FC#UCC (1F(1A)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            A = (RF-E) +DIAN (IF ETA)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         FS11N = -A/RP + GAP1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            FFITE (6,65)
FERMAT(//* *****
CC 10 43
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                IAL = EPS + CELIA
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       A = B/CIAN(THEIA)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               CLLIAL = CELIA/2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                ALINI - ALPHINIZ
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      7/112 = 711/2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    CAM 1E = GA 11/2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        CELTAD = 0.0
TAL = EPS
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     P.S.1F = -A/RP
EPSE = EPS/2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              14CC = 14C/2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       MK 17 E ( 6,65 )
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06 FSF 4A11//
43 CENTINE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                B1 = KC
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             L = RL
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CC 76
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2033 0.024

PAGE 0002

16/15/04

CATE = 91135

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FCRIRAN IV C LEVEL

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PACE CO03
                                                                                                                                                 10 x1 = RE#(-CSIN(TFETAB*,5-ALPHA)-ALFH##CCS(TFETAE*,5-ALPHA))
Y1 = HB*(JCOS(TFETAE*,5-ALPHA)-ALPFA*DSIN(TFETAE*,5-ALPHA))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                pC = -(RP-82)*CCIN(TAU+PSI)*(RF*PSI+&Z)*DCC(SITAL+PSI)
vL = (FP-82)*CC(S(TAU+PSI)*(RP*FSI+AZ)*CSIN(TAU+PSI)
v1 = (FP-E)*JCC(S(TAU+FSI)*(RP*PSI+A)*CSIN(TAU+PSI)
IF(I,EC,I) GG TC EC
 LATE = 81135
                    ALFINC = ALPHF/2
hfiff(c,5) ALINC,ALFIND
FCHMAI(//'',ALPHIN =',F9,5,5x,*ALFFFIN =',F9,5)
                                                                                                                                                                                                                                                                                                                                                                                                                 PSI = PSIIA
PSIC = PSI/2
WEITE(6,6) PSIU,PSIFO
FCFMAT(7/' ',PSIIN =',F9.5,5X,'PSIFIN =',FC.5)
                                                                                                                                                                                                                                                                                                                                                                      TRENCATE VALUE OF YTHICH
                                                                                                                                                                                            IF {AlPFA .cc. aL FHF)GC 1C 2C

$LPH$ = ALPFA + CELA[*Z]

I = [*]

IFFC = I
                                                                                                                                                                                                                                                                        REVERSE DRUER TC CESCEND
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     VI = V141003

IVI = [FIX(VI)

1EMP = V(1) = IVI

IFITEMP_LE_01 GC IC SC
                                                                                 INVCLUTE PLCT
                                                                                                                                          ALPHA = ALPHIN
                                                                                                                                                                                                                                                                                                       20 UC 16 K=1.1

J = 1 - (K-1)

X2(J) = X1(K)

Y2(J) = Y1(K)

16 CCN 11NLE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     Y1 = Y1*1CCC.
IY1 = IFIX(Y1)
                                                                                                                                                                        1A = (1)1A
7X = (1)1X
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 T + T = T
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FCRIFAN IV G LEVEL
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C1117
C1118
O1150
O121
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0103
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PHIRC = CAFCOSIRE/RO)
PHIELD = PHIRL/Z
BEIA + DAFCCSIRE/RC)-(APICS/IZ*RP)+CIAFIPFIFC)-FFIRC-DIANITFEIF)+
                                                                                                                                                                                                                                                                                                              500 YCFIN = (RF-82)+CCCS(1AL+PSIF)+(RP+FSIF+A;)+DSIN(TAL+PSIF)
YFIN = YCFIN - RC
    LCONICA (F. 111A) V. IN PERMYZIK)
21AEL - KV - 3005
J. E. 1.
                                                                                                                             SEARCH FOR MAX VALUE IN EACH ROT
                                                                                                                                                                                                     15(x(1,1),GT.x2(K)) x2(K) = X(1,1)
                                                                                                                                                                                                                                                               CETERMINATION OF CISTANCE RV
                                                                            IF(PSIC_LE.PSIFD) GC TC SCC
J = J + I
JFIA = J
GC TC 25
                                                                                                                                                                                                                  V2(K) = V(1)/1000-D0
1F(V2(K)-LE-YFIN) GU FC 21
1 = 1 + 1
GC TC 5C1
                                                                                                                                                                                       x2(K) = -10.000
CC 15 J=1,JFIN
                                                                                                                                                                         K = I + IPREV
KFIN = K
1A1 = (1) 4 05
                                                                                                                                                                                                            CCNT INLE
                                                                1 50
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0124
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C14c
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C153
C154
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0.165

PACE 0004

16/15/04

CATE \* 81135

Y I V

FCRIMAN IV G LEVEL 21

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P.F.G.E 0005
  16/19/04
                                                                                                                                                                                                                                                                      THIS WILL PRINT THE FULL TCCTH PRCFILE: K, XZIKI, YZIKI
                                                                                                                                                                                                                                                                                                                                                                                                                            61 JC 63 K=1,KFIN

WHITE(6,64) K,VZ(K),XZ(K)

64 FCRIATE K=*,14,5X,*YZ(K) =*,FIC.5,5X,*7Z(K) =*,FIO.5)

63 CKINCE

979 IF(ISTCP.GI.Q) GC IC 99
  LATE = 81135
                                                                                                                                                                                                                                           CETERPINATION OF LEWIS AND ACMA FACIORS
                                                                                                                         KFIN = KFIN - BEGIN

1C4 DC 1C5 J=1.KFIN

K = BEGIN + J - 1

XFAR = DSGRII(R1-CAES(Y2(K1))/CCNB)

C1FF = DSGRII(R1-CAES(Y2(K1))/CCNB)

1F(UFF = DABS(X2(K1)) - XPAR

1F(UFF = LE = D = 0) GC TC 1C5
                                                                                  DETERMINATION OF TANCENCY FOINT
                     102 JIFF1 = 72(1) - 511R1
JF(DIFF1-LE-0-C) GC 10 1C3
                                                                                                                                                                                                  LCAE = CCAB - CELCCN
                                          1 = 1 + 1
CC 10 162
                                                                                                                                                                                         CCN TINLE
FORTHAN IV G LEVEL 21
                                                                                                                 103
                                                                                                                                                                                        105
                     0166
C167
C168
0169
                                                                                                                C17C
0171
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C174
C175
C176
0177
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                                                                                                                                                                                                                                                                                                                                                                   CL#7
0184
C185
                                                                                                                                                                                                                                                                         0 180
0181
C182
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FGKTRAN IV C LEVEL

1000

TITLE- CEKKER'S ALGCRITER

C INITIALLY A 1S SET ELLAL IC C.

INITIALLY A 1S SET ELLAL IC C.

LSE ESELLINES AND ICL=1.C-10.

C SUBRICITIVES AND ICR FUNCTION SLEPROCRAMS FEGUIREC- F(x)

CCMMENTS

IMPLICIT REAL+8 (A-H,C-Z)

K( LN 1=-1 F( A=F ( A) F( E=F ( P)

1C(=1(C)

-843-

IFIDABSIFCB). LE. CAESIFCCI)CC TC 7

IF ((FCE\*FUC) +GT.O.DU )GC TC 6

KCLN 1=KCLN 1+1

LrkkiR=B

F [ B=F C C

おりょう F.C.=1

4200 4200

CAFFCL

PURPOSE- TO FIND A ROOT OF A NOA-LINETR ALCTERATO EQUATION FIX)=0.

06.SCRIFTION OF PARAMETERS

B = THE LATEST ITERATE AND CLOSEST AFFFOXIMITION TO THE ROOT.

A = THE PREVIOUS OF AN ULITH ITERATE.

16/19/04

PAGE CCC1

1F (JAES (E-C) - LE - Z - CO + ICL | ) PETURN 1F (CABS (F CE - F CA) - C 1 - 1 - C - 1C 1 RETURN 1F (F CUNT - G C - L I M 11) GC 1 C 1 Z IFIFCS.EQ.U.DO)RETUFA TLLI=2.EQ\*EPS\*CAPS(B1+G.5GG+TCL

L 1= 6- ( ( 0- 4 ) \*F ( A 1/1 F ( E -F C A )

66 = C 1

9700

9200 00 30 00 31

LEPRENES f ( k = f ( c )

00.12 00.33 00.34 00.35 35.00 1500

4 = 1

111=111

AF 178 (6.15)

CATE = 81135

USFRER

FURTRAN IV G LEVEL 21

0041

SICF
15 FLRPATIT--, "EFRCP IN INPLI ESTA. FIR) HAS THE SAME SIGN AS",
1 " FIC.)";
13 FCRMATIT--, "NUMBER OF ITERATIONS HAS EXCECCE LIMIT. STOP COMPLTA
11 ILLNS.')
1 hos.')

0043 9400

G-8 -150-

FORTHAR IN C. LEVEL 41	t :1	<b>L</b>	<b>EATE</b> = 81135	16 / 19 / 04	PAGE CCC1
1000 2000 1000	37.0	OLUULI FRECISIEN FUNCTIEN FEX) IMPLICIT KEALOB (A-F,C-2) ELWEN MP,G,KO,THETA			
<b>+</b> cno	F1-1 H1-1 1447	F=LATFA [CSURT(X402-(RP-B 1002)/(FF-B 1)-DSCF1(X402-(RP-B )002)/RP-LTHLIF+(RP-B )001/RP-DSCRI(X002-R[002)/RB+[ATAN(DSQRI(X002-R[002)/RB+[ATAN(DSQRI(X002-R[002)/RB+[ATAN(DSQRI(X002-RB002)/RB)]	-B ))-DSGF1(x**2-(RP-6 [(x**2-RE**2)/RB+[AIAN	8 )**21/8P	
9313 9313	RETUFN ENE	JFN			

HC = 0.04000 FD = 1. (( CC C . AFTUS = 1. E4 €CC BEFF = 1.:1500 THETAE = 20.0000 A = 36.CCUCC

KAUU = 1.425CC

22333 \* = 8432 CELCCA = 0.C0100 GELFS1 = C.125CC DELAL = C.25CCC

\*\*\*\*\* TUCIF IS NOT UNDERCUT \*\*\*\*

1AU = 14.97144 DELTA = -8.62052 CANT = C.0 EPS = 23.7919E THETAB = 7.54352

RF = 17.11373

RE = 16.91447 RC = 15.425CC

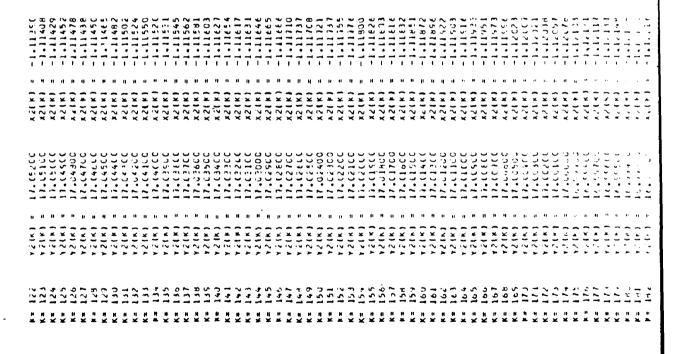
RP = 18. CCCC

ALPLFIN = 32,35465 ALPHIN = 8.82052 PSIFIN =-10.62580 PSIIN . J.J (AP10 = 0.60383 THETAC = C.85052 BEIA = 28.56269 FSIFU = 29.+5321

RV = 19.258:1

YAGMA = 6.36221 YLEFIS = 0.36852 EX = C.55276 CCAB = 1. ECS XPER TAN = 1.132784 X2(KTAN) =-1.13277C Y2(KIAN) = 10.53700 TANCENCY K =237

NRITE =1



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| Mar. 244 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.13556 |
| Mar. 246 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.13556 |
| Mar. 257 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.13556 |
| Mar. 258 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 259 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 250 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 250 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 251 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 252 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 253 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 254 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 255 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 256 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 257 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 258 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 259 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 250 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 250 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 251 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 252 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 254 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 255 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 256 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 257 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 258 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 259 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
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| Mar. 250 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
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| Mar. 250 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 250 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 250 | V.2(N) | M. [6.520CC | X.2(N) | M. -1.1356 |
| Mar. 250 |
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KH 427 Y2(K) H 16.747CC X2(K) H -1.22433
KH 429 Y2(K) H 16.749CC X2(K) H -1.22433
KH 431 Y2(K) H 16.749CC X2(K) H -1.22761
KH 433 Y2(K) H 16.749CC X2(K) H -1.22761
KH 434 Y2(K) H 16.749CC X2(K) H -1.22761
KH 435 Y2(K) H 16.749CC X2(K) H -1.22761
KH 435 Y2(K) H 16.729CC X2(K) H -1.23161
KH 436 Y2(K) H 16.729CC X2(K) H -1.23161
KH 437 Y2(K) H 16.729CC X2(K) H -1.23161
KH 440 Y2(K) H 16.729CC X2(K) H -1.23161
KH 441 Y2(K) H 16.729CC X2(K) H -1.2463
KH 442 Y2(K) H 16.729CC X2(K) H -1.2463
KH 444 Y2(K) H 16.729CC X2(K) H -1.2463
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KH 455 Y2(K) H 16.729CC X2(K) H -1.26633
KH 456 Y2(K) H 16.729CC X2(K) H -1.26633
KH 457 Y2(K) H 16.729CC X2(K) H -1.26633
KH 458 Y2(K) H 16.729CC X2(K) H -1.26633
KH 459 Y2(K) H 16.729CC X2(K) H -1.26633
KH 459 Y2(K) H 16.729CC X2(K) H -1.26633
KH 455 Y2(K) H 16.729CC X2(K) H -1.26633
KH 455 Y2(K) H 16.729CC X2(K) H -1.26633
KH 456 Y2(K) H 16.729CC X2(K) H -1.26633
KH 457 Y2(K) H 16.729CC X2(K) H -1.26633
KH 458 Y2(K) H 16.729CC X2(K) H -1.26633
KH 459 Y2(K) H 16.729CC X2(K) H -1.26633
```

```
TAU = 28.35393
                                                                                                                                            CONR = 4.00000
                                                                                                                                                                                                                                                                                                                                                                                 THETAD = 2.54104
                                                                                                                                                                                                                                                               DELTA = 0.0
                                                                                                                                                                                                                         RF = 5.65550
                                                                                                                                            CELPSI = 0.50000 NELCON* 0.0010C
                                                                                                                                                                                                                                                               EPS = 28.35393
                                                                                                                                                                                                                                                                                                                                                                                  CAPTO = 0.62089
                                        RC = 0.30000
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        YAGMA = 0.23640
                                                                                                                                                                                                                         RO = 7.00000
PD = 1.00000
                                                                                                                                                                                                                                                               3AMT = 26.16565
                                                                                                                                                                                                                                                                                                     ALPHFIN = 42.15911
                                     CAPTCS = 1.57079
                                                                                                                                                                                                                                                                                                                                            PSIFIN =-17.19548
                                                                                                                                                                                                                                                                                                                                                                                  BETA * 33.80518
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        YLEHIS = 0.22964
BEFF # 1.05260
                                                                                                                                                                                **** TCCTH IS UNDERCUT ****
                                                                                                                                                                                                                         RB = 5.63816
                                                                                                                                            DELAL = 0.25000
                                                                                                                                                                                                                                                               THETAE = 16.70787
                                     THE TAC = 20.00000
                                                                                                                                                                                                                                                                                                     ALPHIN = 4.49751
                                                                                                                                                                                                                                                                                                                                            PSIIN = 8.97017
                                                                                                                                                                                                                      RP = 6.00000
                                                                                                                                                                                                                                                                                                                                                                                 PHIRC = 36.34621
                                                                           KACC . 1.00000
                                                                                                                                                                                                                                                                                                                                                                                                                        RV = 6.78532
N = 12.00000
```

NRITE =1

CONB = 2.903

X2(KTAN) =-0.78C016 XPAR TAN = 0.780030

Y21KTAN) = 5.01900

TANGENCY K =729

EX # 0.34446

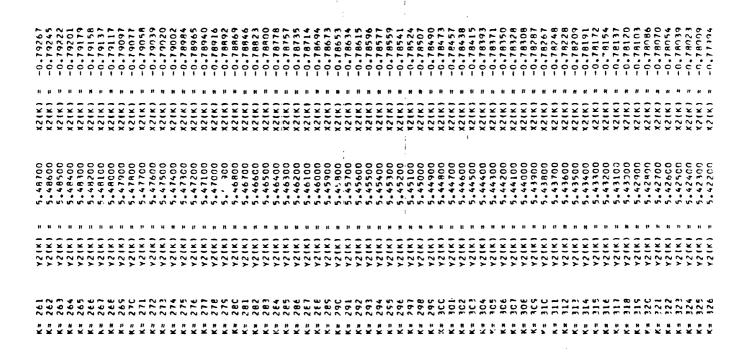


		-		
7185	0.735 0.735 0.744 0.744 0.750	75641	00000000000000000000000000000000000000	
X2(K) X2(K) X2(K)	77777777	72222222222		
1590	1064			5.83276 5.83276 6.818330 6.818330 6.818330 6.818330 6.818330 6.818330 6.818330 6.818330 6.818330 6.818330 6.818330 6.818330 6.81830 6.
2222	77777777	7777777777777777	777777777777777777777777777777777777777	
	**************************************			

G-21 -163-



| 1969 | V21K| | 25.55300 | X21K| | 2 -0.800494 | X21K| | 2 -0.800



			•
			0.7722 0.7718 0.7718 0.7716 0.7716 0.7777 0.7777 0.7777 0.7777
22222222222222222222222222222222222222	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	22222222222222222222222222222222222222	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
- 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0		2000 2000 2000 2000 2000 2000 2000 200	
55555555555555555555555555555555555555	555555555555 		
	~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~	**************************************	7 / 2 / 2 / 2 / 2 / 2 / 2 / 2 / 2 / 2 /
		2222222222222	***************

 K. 459
 YZIK)
 5.28930
 XZIK)
 -0.76314

 K. 460
 YZIK)
 5.28600
 XZIK)
 -0.7629

 K. 461
 YZIK)
 5.28600
 XZIK)
 -0.7629

 K. 462
 YZIK)
 5.28600
 XZIK)
 -0.7629

 K. 464
 YZIK)
 5.28600
 XZIK)
 -0.7629

 K. 466
 YZIK)
 5.28600
 XZIK)
 -0.7629

 K. 476
 YZIK)
 5.28600
 XZIK)
 -0.7629

 K. 476
 YZIK)
 5.28600
 XZIK)
 -0.7629

 K. 476
 YZIK)
 5.28600
 XZIK)
 -0.7629

 K. 477
 YZIK)
 5.27700
 XZIK)
 -0.7629

 K. 478
 YZIK)
 5.27700
 XZIK)
 -0.7621

 K. 478
 YZIK)
 5.27700
 XZIK)
 -0.7621

 K. 489
 YZIK)
 5.27600
 XZIK)
 -0.7621

 K. 484
 YZIK)
 5.26600
 XZIK)
 -0.7621

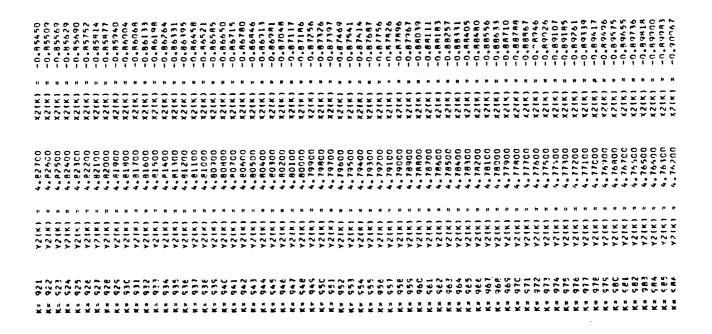
 K. 484
 YZIK)
 5.26600
 XZIK



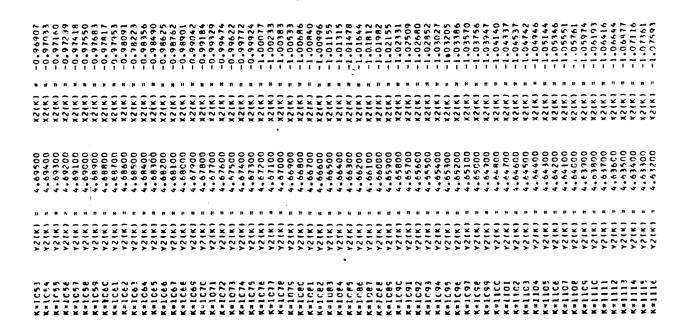
ر د د د د د		,,	0683	X2(K) =	0.7679
<del></del> .	7		•		
	7	*	. 0880	X2(R) =	0.7681
	~	,,	.0870	* (X)cX	0.7692
	2	! !	.0860	~;	0.7693
	7 7	H H	0.687	;;	7685
	2	ı	0830	: ~	0.7686
	~	"	. 0820	~	0.7688
	~ ?	<b>,</b>	0180.	2 5	7490
	7 2	! 	0440	7 7	7691
	7	н	.0780	7	0.7693
	~	n	.0770	2	.7694
	~ ~	H 1	.0760	5	0.7696
	7	. 4	0740	5 2	7699
	- 2		0130	21	0.1700
	2	#	.0720	2	.1702
_	2		0110	7	0.7704
	2	*	0100	7	0.7705
	2 :		0690	2	1011-0
	7 0		06.40	2.5	72.0
. ~	7	, ,,	0990	7	7112
	7		.0650	2	0.7713
•	7	*	.0640	2	0.7715
S.	2	,,	.0630	7	0.7716
•••	2	H 1	.0620	7	0.7718
- u	7 7	) pi		7 7	0.7721
	2 .		0530	7	0.772
· U	72(K)	,	5.05800	X2(K) =	-0.77243
_	2	n	.0570	2	0.7725
~ •	2 5		.0560	2 5	0.1727 0.547
	7	. #	0540	; ;	0.7731
· •	7		.0530	7	0.7732
•	7		.0520	7	0.7734
~ (	2	#	0150	2	0.7736
<b>.</b>	7 7		0000	7 7	0.7760
	7		0480	7	0.7742
_	7		0470	7	.7744
2	7	н	0460	21	.1746
<b>.</b>	7	H	0420	7	-7748
•	~ ?		0440	2	1150
٠.	; ;	. 4	0420	, ,	7753
, ~	7		2410	21	7755
	7	H	0400	7	.1756
·	7	#	0340	7	.1758
۰	~	ĸ	0380	2	.7760
	~		07.5	21	-7762
~ •	~ ;	4 (	2960	7	25.1.
n .	7		26.00	7	7768
				, ,	7770
٠.	7		0370	; ;	1112
. ~		н	10	7	7774
			300	7	0.7776
		H	290	2	.777A
c		*	280	2	7 7 R
_		*	270	X21K) =	. 779
~	V2(K)		260	x2(K) =	. 778

| No. | No.





K. 5EF V2IK) = 4.76100 XZIK) = -0.90132
K. 59E VZIK) = 4.75600 XZIK) = -0.004410
K. 50E VZIK) = 4.75600 XZIK) = -0.041184
K. 50E VZIK) = 4.77600 XZIK) = -0.04184
K. 50E VZIK) = 4.77600 XZIK) = -0.04184



K=111G Y2(K) = 4.62700 X2(K) = -1.08324
K=1121 Y2(K) = 4.62700 X2(K) = -1.08581
Y2(K) = 4.62700 X2(K) = -1.09116
K=1122 Y2(K) = 4.62700 X2(K) = -1.09116
K=1123 Y2(K) = 4.62700 X2(K) = -1.09116
K=1124 Y2(K) = 4.62700 X2(K) = -1.09116
K=1125 Y2(K) = 4.62700 X2(K) = -1.09977
K=1126 Y2(K) = 4.62700 X2(K) = -1.09977
K=1127 Y2(K) = 4.62700 X2(K) = -1.09977
K=1126 Y2(K) = 4.62700 X2(K) = -1.09577
K=1127 Y2(K) = 4.61700 X2(K) = -1.1167
K=1137 Y2(K) = 4.61700 X2(K) = -1.1167
K=1137 Y2(K) = 4.61700 X2(K) = -1.1167
K=1137 Y2(K) = 4.61700 X2(K) = -1.1167
K=1133 Y2(K) = 4.61700 X2(K) = -1.1167
K=1133 Y2(K) = 4.61700 X2(K) = -1.1167
K=1133 Y2(K) = 4.61700 X2(K) = -1.1167

## APPENDIX H

PROGRAM LEWIS ENVELOPE

```
PAGE 0001
  15/15/27
                                                                                                                                                                                                                                                                                                                                                                                                  hRITE(6,2) K400
2 FORPAT(//* ', 'KA00 = ',F9.5,//)
hRITE(6,4) DELAL, DELPSI, DELCON, CONB
4 FORPAT(//* ', 'DELAL = ',F9.5,5%', 'DELPSI = ',F9.5,5%', 'DELCON* ',F9.5,
15%', CCNB = ',F9.5)
                                                                                                                                                                                                                                                                                                                                                                    I FORMAT('11', 'M a', F9.5,5%, 'BEFF =',F9.5,5%, 'PD =',F9.5)

WRITE(6,3)THETAD, CAPTCS, RC

3 FORMAT(//' THETAD =',F9.5,5%, 'CAPTCS =',F9.5,5%, 'RC =',F9.5)
                                                                               SEARCH FOR CONB (B) THAT GIVES LEWIS PARABOLA WHICH IS TANGENT TO TOOTH PROFILE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 HETAB = CAPTCS+DCOS(THETA)/RB + 2.*IDTAN(THETA)-THETA)
PS = THETAB/2.DO * THETA
HETBD = THETAB/2.
 CATE = 81159
                                                                                                                           PMPLICIT REAL®64A-W.O-2)
COMPON RP.B.RB.THETA
CIPENSION XILBOD, YILBOD, X2(1000) V2(1000)
INTEGER BEGIN,FINISH
REAL XC, XT, VT, VC, VTT, KT, N
                                                                                                                                                                                                                          PEAD(5,51) N.BEFF,PD,TMETAD.CAPTCS.RC
READ(5,52) KADO
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              EALLOW * RP+OSINITHETA)+DSINITHETA)
FREEFF.GI.BALLOW) GO TO 41
FFREEFF.LE.BALLOW) GO TO 42
 Z T T
                                                                                                                                                                                                                                               READ(5.53) DELPSI-DELAL
READ(5.54) DELCON.CONB
READ(5.55) NRIVE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             A = (RP-8)*CTAN(THETA)
CELTAD = 0.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                RS = RP+CCOS(THETA)
RO = RP+KADD/PO
                                                                                                                                                                                                                                                                                                                                                           TRITE (6. 1)N. BEFF. PO
                                             LENIS FNVELOPE
                                                                                                                                                                                                                                                                                                                                                                                                                                                                            # P1/180.
HETA 4 THETAD92
                                                                                                                                                                                                   EAC (5,910) | STOP
                                                                                                                                                                                                                                                                                              51 FORMATICFO.5)
52 FORMATICFO.51
51 FORMATICFO.51
54 FORMATICFO.51
55 FORMATICLI
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     P * N/12.0PD)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        8 = 8EFF/PD
                                                                                                                                                                                                             91C FORMATITI
12 140...
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  C042
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PAGE 0002
 15/15/27
                                                                                                                                                                                                                                                                                                                                                       PRITE(6,70) RP.RB.RO.RF
7C RORMAT(" RP = ",F9.5,5x,"RB = ",F9.5,5x,"RO = ",F9.5,5x,"RF = ",F9.5/)
                                                                                                                                                                                                                                                                                                                                                                                                   brife(0.71) THETBD.GAMTD.EPSD.DELTAD.TAUD
71 PORPAT(/* THETAB **.F9.5,5x,*GAMT **.F9.5,5x,*RPS **.F9.5,5x,*DELT
18 **.F9.5,5x,*TAU **.F9.5,5
                                                                                                                                                                                                                                               RF = DSGRT(DASS((RB*DTAN(THETA)-B/DSIN(THETA)) 402*RB*RB))
6APT = 0.0
BLPHIN = DSGRT(DASS(RF*RF-RB*RB))/RB
IF (ALPHIN-LE-0.001) ALPHIN = 0
ALPHF = 0.7AN(DARCOS(RB/RD))
PSIIN = 0.0
                                                                                                                                                                                                                                                                                                                                     (//....
 CATE = 81159
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 #LFINC = ALPHF/Z
hrite(6,5] Alind,Alfind
format(//* *,*Alphin =*,F9.5,5x,*Alphfin =*,F9.5)
                                                                                                                             (//....
                                                RFECKER(BT.C.1.D-16.1.D-18.100)
ALPHF = DSQRT(DAS(RFERF-RBSRB))/RB
ALPHF = DSQRT(DAS(RFERF-RBSRB))/RB
GAFT = CSQRT(DAS(RFERF-(RP-BEFF)++2)))/RP
PSIF = -A/RP
NITE(8.65)
65 FDRM | 10 43
                                                                                                                                                                                                                                                                                                                                                                                                                                            FORMATI'/ **** TOOTH IS NOT UNDERCUT
                                                                                                                                                                                                                CELTA + B/IRB-OSINITHETA)) - DIANITHETA)
CELTAC = DELTA/2
TAU = EPS + DELTA
 Z
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 02 = 0 - RC+DSIN(THETA)

M2 = A - RC+DCOS(THETA)

ALIND + ALPHIN/Z
                                                                                                                                                                                                        - B/DTAN(THETA)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 INVOLUTE PLOT
                                                                                                                                                                                                                                                                                                                                                                                          CAPTO . GAMT/Z
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       PSIFD + PSIF/2
                                                                                                                                                                                                                                                                                                                                                                               TALC - TAU/2
                                                                                                                                                                                                                                                                                                                           LR [TE(0,66)
                     TAU . EPS
                                                                                                                                                                                                                                                                                                                                     66 FORMATCZ
                                RT = R0
FCRIRAN IV G LEVEL 21
                                                                                                                                                                                                        •
                                                                                                                                                                                                       42
                                                                                                                                                    . . . . . .
                                                                                                                                                                                                                                                                                                                                                                                                                                   00000
                                          66074
66074
66679
66679
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C083

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15/15/27
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 PETA * DARCOS(RB/RO)-CAPTCS/(2*RP)+OTAN(PHTRO)-PHTRO-DTAN(THETA)+
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 EETAD # BETA/2
CAPTO * CAPTCS*RO/RP-2*RO*(DTANIPHIRO)-PHIRO-DTANITHETA)+THETA)
THETAC * CAPTO/12.*RO)
THECO * THETAC/2
RV * RO*(DCCS(THETAC)-DSINITHETAO)*CTANIBETA))
                                                                                              XI = R001-DSINITHETABO.5-ALPHA1-ALPHA0CCOSTHETABO.5-ALPHA1)
VI = R001DCCSITHETABO.5-ALPHA1-ALPHA0CSINITHETABO.5-ALPHA1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     XENUE - (RP-B2) + DECOS (NIT AU+PSI) + (RP+PSI+A2) + DECOS (TAU+PSI) + IRC+ (IRP+PSI+A2) + DECOS (TAU+PSI) + BZ+DSIN(TAU+PSI) + BZ+DSIN(TAU+PSI) + BZ+DSIN(TAU+PSI) + BZ+DSIN(TAU+PSI) + BZ+DSIN(TAU+PSI) + BZ+DN - BZ+DN - BZ+DSIN(TAU+PSI) + BZ+DN - BZ+
   CATE . 81159
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               PRITE(6.6) PSIO.PSIED
FORPAT(//* ', 'PSIEN =',F9.5,5%',PSIFIN =',F9.5)
[ = IPREV + 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   ***** TRUCHOID PLO: *****
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            CETERMINATION OF DISTANCE RV
                                                                                                                                                                                   VIII) # VI

F (ALPHAGE.ALPHF)GD TO 20

ALPHA # ALPHA + DELAL+?

[ * I+1

IPREV # I
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             PSIO = PSIO - DELPSI
PSI = PSIO+2
IF(PSIO-LE-PSIFD) 60 70 500
                                                                                                                                                                                                                                                                                                                                                                                                                                                           REVERSE DRDER TO DESCEND
   MAIN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          21 PHIRG = DARCOS(28/RG)
PHIRGG = PHIRG/2
                                                               BLPHA & ALPHIN
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     1 = [ - (K-1)
x2(1) = X1(K)
Y2(1) = Y1(K)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         CO 16 K+1,1
                                                                                                                                                                 X 1 (1) X
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      CONTINBE
                                                                                                                                                                                                                                                                                                                                                             01 01 09
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    # 45 H
   FCRIRAN IV G LEVEL 21
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 900
                                                                                                    ະ
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         20
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      151
                                                                                                                                                          C117
C118
C119
C120
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                C113
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         6034
6635
6637
6637
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     CIC5
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PAGE 0003

```
VAGPA & 1.200/(DCOS(BETA)/DCOS(THETA)*(1.5D0/EX -
LCTAN(BETA)/(2.005/E/N.)))

MRITE(6.111) EX.YCEMIS.YAGMA

111 FORMAT(//**,*EX =*,F9.5,5X,*YCEMIS =*,F9.5,5X,*YAGMA =*,F9.5)

MRITE(6.110) K,YZ(K),XZ(K),XPAR,CONB

110 FORMAT(//**,F9.6,5X,*YZ(KTAN) =*,F9.5,5X,*XZ(KTAN) =*,

1149.6,5X,*YPAR TAN =*,F9.6,5X,*CCNB =*,F6.3/)

MRITE(6.31) MRITE

31 FORMAT(//**,10X,*MRITE =*,11/*1*)

11 FORMAT(//**,10X,*MRITE =*,11/*1*)
LRITE(6,100) PHIROD, BETAD, CAPTO, THEOD

ICC PORMAT(//* PHIRO = ', F9.5, 5X, 'BETA = ', F5.5, 5X, 'CAPTO = ', F9.5, 5X, 'BETA = ', F5.5, 5X, 'CAPTO = ', F9.5, 5X, 'BETA = ', F5.5, 5X, 'CAPTO = ', F9.5, 5X, 'BETA = ', F5.5, 5X, 'CAPTO = ', F9.5, 7\)

ROT FORPAT(' RV = ', F9.5/)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       THIS WILL PRINT THE FULL TOOTH PROFILED K. 121K), YZTK)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   ::
                                                                                                                                                                                                                                                                                                                                                                                                                                                                DETERMINATION OF LEWIS AND AGMA FACTORS
                                                                                               LOCATION OF POINT V IN ARRAY Y21K)
                                                                                                                                                                                                                                                                                                                                                 XPAR - DSQRT(IRV-DABS(YZIK))/CONB)
CIFFZ + DABS(XZIK)} - XPAR
IF(CIFFZ-LE.O.O) GC TC 109
                                                                                                                                                                                                                                                     DETERMINATION OF TANGENCY POINT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             PLOT
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             INVOLUTE
                                                                                                                                                    109 EX = X2(K)002/(RV-Y2(K))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       YLEMIS = 2.00*PD*EX/3
                                                                                                                                                                                                                                                                                               PEGIN 4 I
MFINI 4 KFIN - BEGIN
CO 105 J=1,KFINI
K = BEGIN + J - 1
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CONS = CONS - DELCON
                                                                                                                                         START # RV - .005
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61 CO 63 K-1,1PREV
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                                                                                                                                                                                                            60 TO 102
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C123
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G125
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C143
C344
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C150
C151
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PAGE 0004

15/15/27

**CATE = 81159** 

PCRIRAN IV G LEVEL 21

	17 13631 0 AT NEWS	-	E		LAIE	LATE = BEISY	15/15/27	PAGE 0005
1613	63	1157 63 CONTINUE						
6512		LRITE(6.57)						
6613	57	FORMATO//	TRECHOID	PLOT	<i>(//.</i>			
2160		KFIN . KFIN - IPREV						
:161		CO 67 1=1.KFIN						
2113		K = I + IPREY						
163		HRITE(6,68) K,Y2(K)	.X2(K)					
1164	9	FORFATI' K= . 14.5X.	.Y2(K) =.,	F10.5.5X	. * X 2 E K	) = (.F10.5)		
3165	4	CONTINUE						
366	666	IF(1510P.61.0) 60 T	66 0					
:167		STCP	٠.					

FCRIRAN IV G LEVEL

1222

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C TAIL THES B AND C BRACKET THE ROOT.
C INITIALLY A IS SET EQUAL TO C.
LSE EPS-1.0-7 AND TOL=1.0-10.
C SUERCUTINES AND/OR FUNCTION SUBPROGRAMS RECUIRED- F(X)
                                                            PURPCSE- TO FIND A ROOT OF A NON-LINEAR ALGEBRAIC BOUATION FIXTED.

CESCRIPTION OF PARAMETERS

E = THE LATEST ITERATE AND CLOSEST APPROXIMATION TO THE ROOT.

A = THE PREVIOUS ITERATE.

C = THE PREVIOUS OR AN OLDER ITERATE.
COUBLE PRECISION FUNCTION DEKKERIB, C, EPS, TOL, LIMITI
                                                                                                                      TITLE- DEKKER'S ALGORITHM
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:08=F0C
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IF((FCB\*FCC),GT.0.D0)C=A IF((FCB\*FCC),GT.0.D0)FCC=FOA 14 ITE(0, 15) CEKKER+8 FOB\*F(B) FOA=FCB 6629 6620 

£028

12 hRITE(6,13)

FEDABSIFOBILE.DABSIFOCISCO TO ?

CEKKER+8

OC=F(C) F((FC8\*FOC).GT.0.00)60 TO 6 OUNT=KQUNT+1

FOA-F(A) FOE-F(B)

KOUNT=-1

IMPLICIT REAL+8 (A-H,O-Z)

FCRIRAN IV G LE		EL 21	DEKKER	CATE = 81159	15/15/27	PAGE 0002
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CATE = 81159

FCRIRAN IV G LEVEL 21

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COUBLE PRECISION FUNCTION F(X)
IMPLICIT REALOB (A-H,O-2)
COPMON RP,BoRG/THETA
FAGATANIDSQRI(X002-1RP-B ))-DSQRI(X002-1RP-B )002)/RP
I-THETA-1RP-B )00TANITHETA )/RP-DSQRI(X002-RB002)/RB+DATANIDSQRI(X002-RB002)/RB+DATANIDSQRI(X002-RB002)/RB+DATANIDSQRI(X002-RB002)/RB+DATANIDSQRI(X002-RB002)/RB)
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KACC = 1.42500

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\*\*\*\* TCDIM IS NOT UNCENCUT \*\*\*\*

TAU = 14.97144 DELTA = -8.82052 EPS = 23.79196 GAMT = 0.0 THETAS . 7.58392

RF = 17-11373

RO \* 19.42500

RB \* 16.91447

RP \* 18.CC000

ALPHIN = 6.82052 ALPHFIN = 32.35465

PSIIN \* 0.0 PSIFIN =-10.62580

PHIRD = 29.45321 BETA = 20.56269 CAPTO = 0.60383 THETAO = 0.89052

RV \* 19.25831

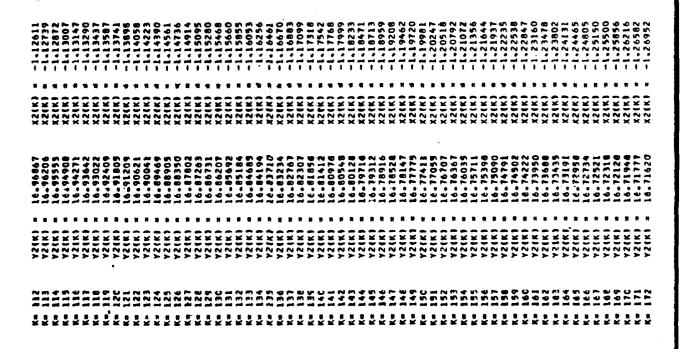
EX = 0.55277 YLEWIS = 0.36851 YAGMA = 0.36221

CONB = 1.809 XPAR TAN = 1.132924 XZ [KTAN] =-1.132901 Y2(KTAN) = 16.93642 TANGENCY K =117

MRITE =1

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PD - 1.00000 BEFF # 1.05260 N = 12.00000 RC = 0.30000 CAPTCS = 1.57079 THETAL = 20.00000

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COMB = 4.00000 DEL CON\* 0.00100 CELPSI = 0.50000 DELAL = 0.25000

\*\*\*\*\* TOOTH IS UNDERCUT \*\*\*\*

RF = 5.65550

RO . 7.00000

RB - 5.65816

RP . 6.00000

. TAU = 28.35393 DELTA = 0.0 EPS = 28.35393 GAMT 4 26.16565 THETAB = 16.70787

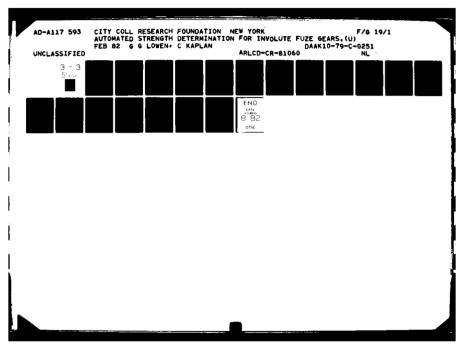
ACPHFIN . 42.15911 ALPHIN = 4.49751

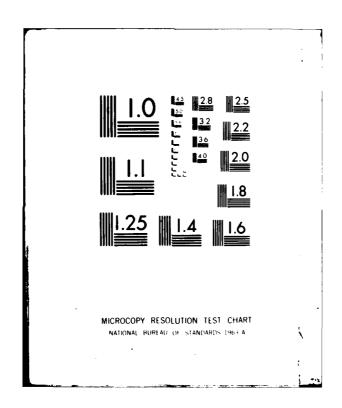
PSIFIN =-17,19548 PSI IN # 8.97017 THETAG = 2.54104 CAPTO = 0.62089 BETA = 33.80518 PHIRC = 36.34621

RV = 6.78532

YAGHA = 0.23643 YLEHIS = 0.22962 EX = 0.34443 CONB = 2.903 XPAR TAN = 0.781767 X21KTAN1 #-0.781725 Y21KTAN1 = 5.01112 TUNGENCY K #177

NR [TE =1





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| March | Marc

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\*\*\*\* TODIM IS UNCERCUT \*\*\*\*

TAU = 28.35393 DELTA . 0.0 RF # . 5.65550 EPS = 28.35393 RO # 7.00000 GAPT 4 26.16565 Re = 5.63816 THE TAE = 16.70787 RP - 6.00000

ALPHFIN = 42.15911 ALPHIN = 4.49751

PSIFIN =-17.19948 PSITH - 8.97017

THETAU = 2.54104 CAPTO = 0.62089 BETA = 33.80518 PHIRC = 36.34621

RV . 6.78532

VAGMA = 0.23635 VLEHIS = 0.22965 EX = 0.34447

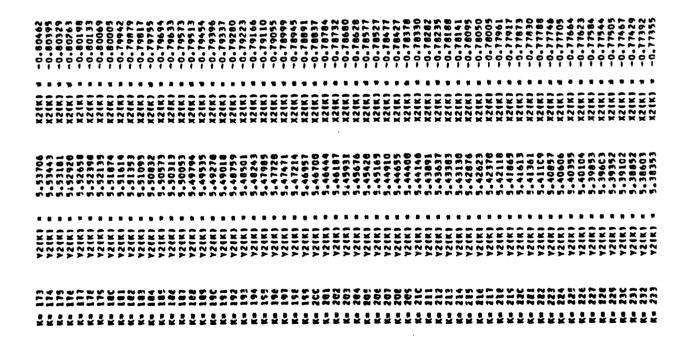
CONB = 2.903 XPAR TAN = 0.177909 X2(KTAN) --0.777906 Y2(KTAN) = 5.02859 TANGENCY K =385

MRITE =1

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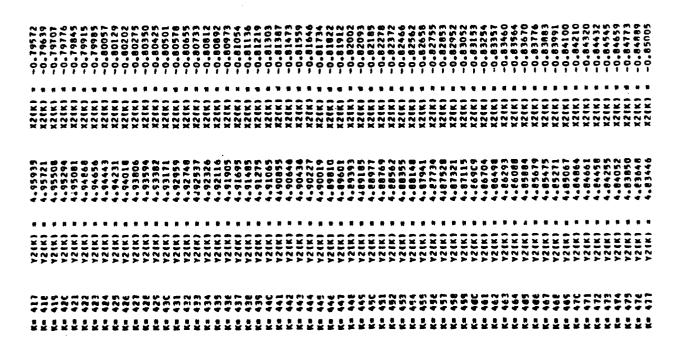


K) 4 -0.81490 K) 4 -0.81836 K) 8 -0.81880		2618-0-	2010-0- * I	10-8703	1 = -0.8705	1 = -0.8207	10.8210	KI = -0.8212	K)0.8213	K)0.8215	K) 4 -0.8216	K) # -0.8217	K)0.8218	K) # -0.8219	K) 4 -0.8220	K10.8220	KI = -0.8220	K)0.8220	1 4 -0.8220	1 4 -0.8220	1 = -0.8219	10.6219	10.8218	10.8217	10.8217	10.8216	10.0215	90-8213	7178-0 1	0128-0- 7 1	P -0.8209	10.6207		0619.0- + 1	1 = -0.8163	1 = -0.8175	10.8167	10918-0- = 1X	2610°0- # 1	. a -0.8137	10.8130	1 = -0.8123	10.8115	-0.8108	1018-0- * 6	\$608°0 1			100°01		PC08-0- # F
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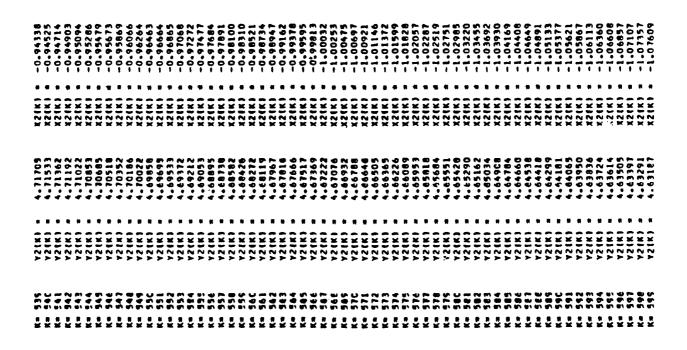




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